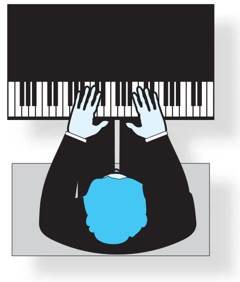
8.2 WAVE INTERACTIONS



Objectives

- Describe what's meant by interference of waves.
- Distinguish between constructive and destructive interference.
- Use the superposition principle to find the resultant wave produced by two interfering waves.
- Explain how a standing wave is formed.
- Define natural frequency.
- Explain how forced vibrations or oscillations can produce resonance.

When two or more waves *of the same type* are combined at the same place at the same time, the result is called **interference**. You can see wave interference if you drop two stones into water. The water waves produced by the stones spread and eventually overlap. A two-dimensional **interference pattern** is produced when the two periodic water waves occupy the same region at the same time.

INTERNET connection

To find out more about wave interactions, follow the links at **www.learningincontext.com.**

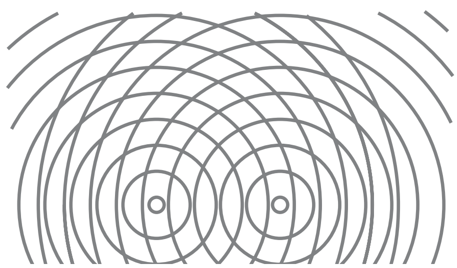


Figure 8.8
Overlapping water waves produce an interference pattern.

The Superposition Principle

Suppose each end of a stretched rope is given a single up-and-down shake, producing identical wave pulses. The pulses travel toward each other, and when they meet they interfere (see Figure 8.9.) At the moment when the peaks of the pulses coincide, the amplitude of the resultant pulse is the sum of the amplitudes of the individual pulses. The displacement of the rope is greater than the displacement caused by either individual pulse. This effect is called **constructive interference**. Notice that each pulse reappears and continues moving in its original direction as if the other pulse did not exist. The properties of a pulse (amplitude, wavelength, speed, and frequency) are unaffected by the presence of another pulse.

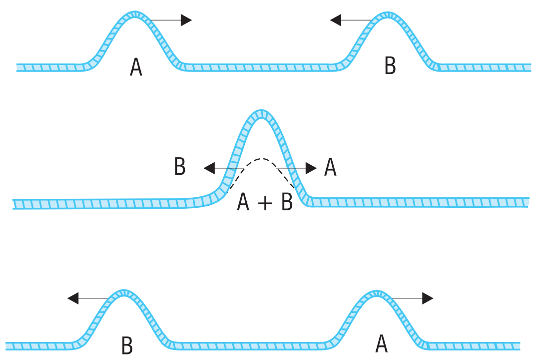


Figure 8.9
Constructive interference of two wave pulses in a stretched rope

Now suppose one of the pulses is inverted, as in Figure 8.10. (One person gives the rope a down-and-up shake.) When these pulse amplitudes coincide, the resultant is still the sum of the amplitudes of the individual pulses. But this time the sum is zero. The displacement of the rope is less than the displacement caused by either individual pulse. This effect is called **destructive interference**. Once again, each pulse reappears and continues moving in its original direction as if the other pulse did not exist.

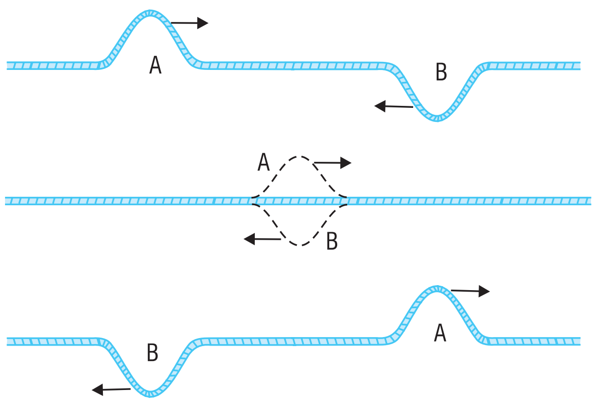
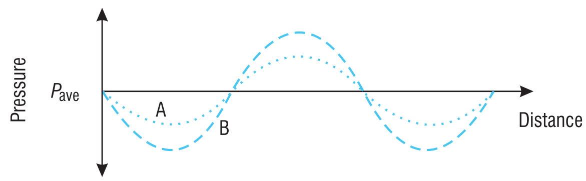


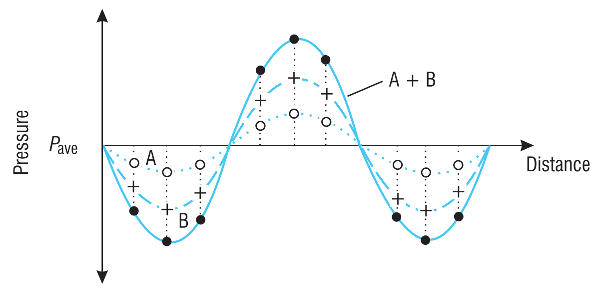
Figure 8.10
Destructive interference of two wave pulses

When waves interfere as in Figures 8.9 and 8.10, you can add the displacements produced by the individual waves to find the displacement of any point in the rope at any given time. The principle of adding displacements of overlapping waves is called the **principle of superposition**. The principle holds for most types of waves, including wave pulses and periodic waves in a stretched rope, water waves, sound waves, and electromagnetic waves. The principle can be used whenever the properties of wave pulses or periodic waves are not affected by the presence of other waves.

Example 8.6 Constructive Interference of Two Sine Waves

At one instant in time, two sound waves, A and B, of the same wavelength interfere as shown below. Sketch the resultant wave.





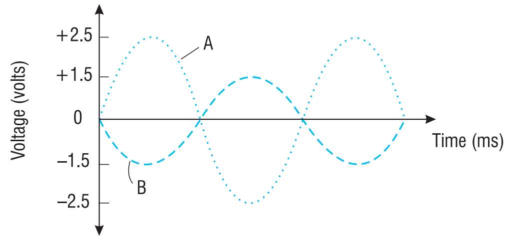
Solution: Using the superposition principle, add A and B. Draw several vertical guidelines from the average pressure line to each wave, as shown above. For each guideline, use segment addition: The segment length to wave A plus the segment length to wave B equals the segment length to the resultant wave (A + B). Make a dot to show the endpoint of each segment. Sketch a sine wave through the dots.

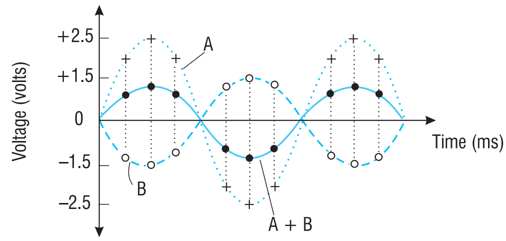
In Example 8.6, notice that the pressure in the resultant wave (A + B) is greater than the pressure in either wave A or wave B. These two waves interfere constructively.

In the example on the next page, two waves interfere destructively. In this case, the resultant wave has an amplitude between the amplitudes of the original waves.

Example 8.7 Destructive Interference of Two Sine Waves

Two voltage signals, A and B, exist simultaneously across a resistor. The signals have the same wavelength but different amplitudes. As shown below, the signals are *out of phase*—one increases while the other decreases. What is the amplitude of the resultant wave?





Solution: Add the two waves using superposition and the same procedure as in Example 8.6. The voltage in wave B is always opposite in sign to the voltage in wave A. Therefore, the amplitude of the resultant is the difference between the amplitude of A and the amplitude of B.

$$\begin{aligned}\text{Amplitude of } (A + B) &= \text{amplitude of A} - \text{amplitude of B}\\ &= 2.5\text{ V} - 1.5\text{ V}\\ &= 1.0\text{ V}\end{aligned}$$

The amplitude of the resultant wave is 1.0 volt.

Wave Reflections

When a wave is reflected, it changes direction. A wave traveling on a stretched rope can be reflected if the end of the rope is tied securely to a wall. Figure 8.11 shows a reflection of a wave pulse. An incident wave I travels to the right, the wave is reflected at a wall, and the reflected wave R travels to the left.

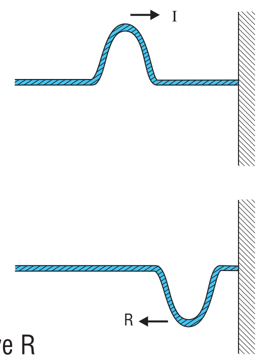


Figure 8.11
An incident wave I and a reflected wave R

Notice that the reflected wave changes direction but the wavelength does not change. Notice also that the reflected wave is inverted. A crest becomes a trough, and a trough becomes a crest.

Suppose the incident wave is periodic instead of a pulse. The reflected wave will be periodic also. In this case, the incident and reflected waves will be present in the rope at the same time, and they interfere. This interference can form a *standing wave* in the rope.

Standing Waves

Sound from a guitar or violin string is created by **standing waves** in the string. These waves are a result of reflections at the fixed ends of the string. Waves travel back and forth between the ends of the string, constantly being reflected and interfering with each other. But the string cannot move at the fixed ends. Therefore, destructive interference must always occur at the ends.

Figure 8.12 shows a standing wave in a string at three instants in time. This standing wave has a wavelength equal to the length of the string. For this wavelength, there is one point in the string (in addition to the ends) that is stationary. This point is called a **node**. The first illustration is at a time when the incident and reflected waves have their crests and troughs overlapping. We say these incident and reflected waves are **in phase**. When two waves are in phase, they constructively interfere. The second illustration is at a time when the incident wave crest overlaps the reflected wave trough, and vice versa. We say these two waves are **out of phase**. Two waves that are out of phase interfere destructively. Are the incident and reflected waves in phase or out of phase in the third illustration?

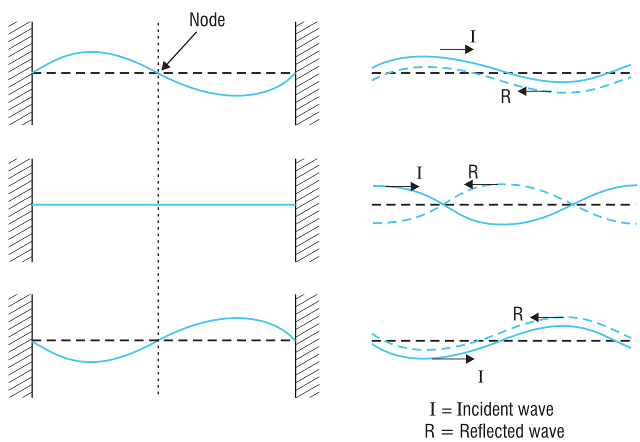


Figure 8.12
A standing wave is produced by interference between incident and reflected waves.

You can make standing waves in a stretched rope or length of flexible tubing. Tie one end of the rope securely to a support. Shake the other end at a low frequency until you see a standing wave with the length of the rope equal to one-half wavelength (see Figure 8.13a). If you double the frequency, you will create a standing wave whose wavelength equals the length of the rope (see Figure 8.13b). If you triple the original frequency, the standing wave will have a wavelength of 2/3 the length of the rope (see Figure 8.13c).

Suppose a rope has length L. A standing wave exists in the rope if incident and reflected waves cause destructive interference at both ends of the rope. This happens only for certain wavelengths λ. As shown in Figure 8.13, a standing wave can exist only if the length of the rope is a multiple of $\lambda/2$. In other words $L = \lambda/2, \lambda, 3\lambda/2, 2\lambda$, and so on. If you solve for the possible wavelengths of a standing wave, $\lambda = 2L, L, 2L/3, L/2$, and so on.

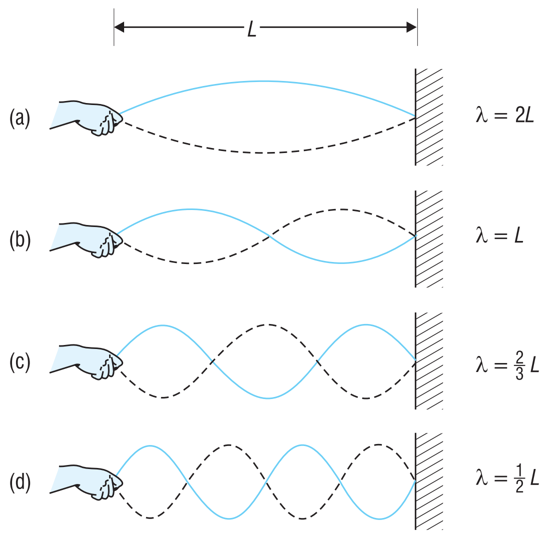


Figure 8.13
The first four possible wavelengths of a standing wave in a stretched rope of length L

Other types of waves also can occur as standing waves when there are reflectors at appropriate distances. A drumhead supports standing waves because it is fixed around its perimeter. Displacement travels as a wave in two dimensions through the drumhead, and the wave is reflected at the perimeter. Vibrating air columns in flutes, organ pipes, and other musical instruments are standing sound waves. Sound waves are reflected at the ends of each musical instrument. Laser light is produced from standing light waves. In a laser, light is reflected by mirrors. Microwave ovens use standing microwaves to deposit energy in food (mostly water) molecules. The inner walls of the oven reflect microwaves. You will learn more about light, microwaves, and other electromagnetic waves in the next chapter.

Applications of Wave Interference

Interference is especially useful in light-wave technology (optics) and sound-wave technology (acoustics).

When light waves of similar frequency and wavelength interfere, they produce a series of bright and dark regions called an *interference pattern*. The bright regions, or *bands*, are a result of constructive interference. The dark bands are a result of destructive interference. One application of interference patterns is the use of optical flats to determine the flatness of machined parts. An optical flat is a piece of perfectly flat glass about ¾ inch thick. It is placed on the machined surface to be checked, and light is directed toward the surface (see Figure 8.14). Some of the light is reflected by the machined surface; some is reflected by the glass.

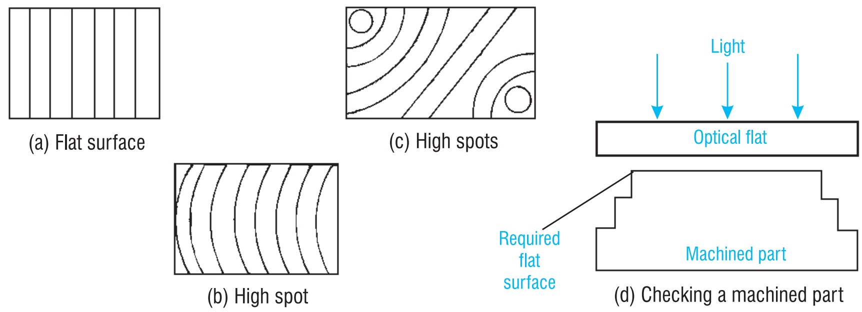


Figure 8.14
Interference patterns indicate the flatness of machined parts.

Since the light from the work surface travels a little farther, the waves are reflected out of step—that is, out of phase. This out-of-phase condition creates an interference pattern. The interference pattern is seen by the machinist, looking down at the surface of the part under the flat. By properly interpreting the interference pattern, the machinist can infer the relative flatness of a machined surface. The drawings in Figure 8.14 give you a sample of some results seen by a machinist checking for flatness.

The interference pattern in Figure 8.14a indicates to the machinist that the part is flat within a wavelength of light. Figure 8.14b (where the bands are curved) indicates that a part is high in the middle. Figure 8.14c (where the bands have swirls in them) indicates that the part has several high spots. Because the wavelength of light is very short (less than 1/1000 of a millimeter), accurate measurements of high or low spots on the surface of the machined part can be made. Then the machinist knows how much more grinding or polishing is needed to make the part flat.

The interference of sound waves also produces patterns that can be controlled. In concert halls and recording studios, reflections (or echoes) interfere with the original sound. Interference is reduced by using heavy drapes, foam, or other absorbing material to eliminate reflections.

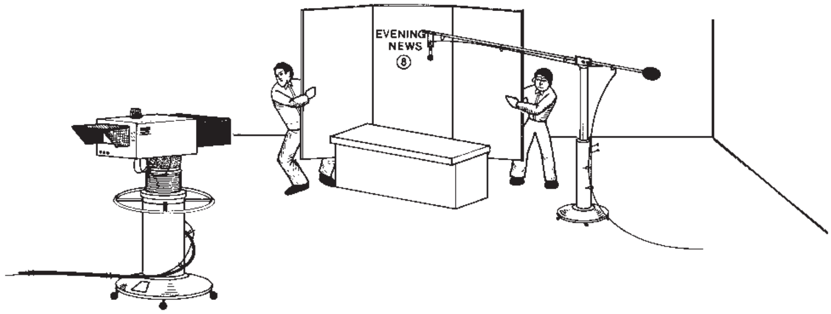


Figure 8.15
Controlling sound waves in a studio

Two sources of sound waves that are at nearly the same frequency (or wavelength) will interfere constructively and destructively in a regular pattern, just as in a standing wave. You hear this pattern as pulsations, or variations in loudness, of the combined sound. The variations in loudness are called **beats**. The beat frequency is much lower than the frequency of either source of sound. The beat frequency equals the difference between the frequencies of the two interfering waves.

For example, suppose a violinist bows a single string, creating a standing wave of frequency 440 Hz (this is the concert pitch of A above middle C). A second violinist bows a note at a frequency of 442 Hz. The combined sound heard by the violinists will have a frequency equal to the *average* of the two frequencies, or 441 Hz. The violinists will also hear a beat, with a frequency equal to the *difference*, or 2 Hz (2 beats per second). If the second violinist slowly reduces the tension in his violin string, the beats will get farther apart, and finally disappear when the two frequencies are the same.

Refer to Appendix F for a career link to this concept.

Natural Frequency

The **natural frequency** of an object is a single frequency at which the object vibrates or oscillates when it does so on its own. Think about a simple pendulum, like a grandfather clock or a child's swing. If the swing is set in motion, it moves back and forth at a certain frequency. If you stop the swing and then start it again, it moves back and forth at the same frequency. As long as you don't change the length, every time you start a pendulum swinging, it oscillates at the same frequency. That's its natural frequency.

Or think about a vibrating spring. One end of the spring is attached to a ceiling. A weight is hung on the other end. The spring is pulled down and let go. It then bobs up and down. It does so at a single frequency—its natural frequency. As long as you don't change the weight, every time you pull the spring down and let it go, it oscillates at the same frequency.

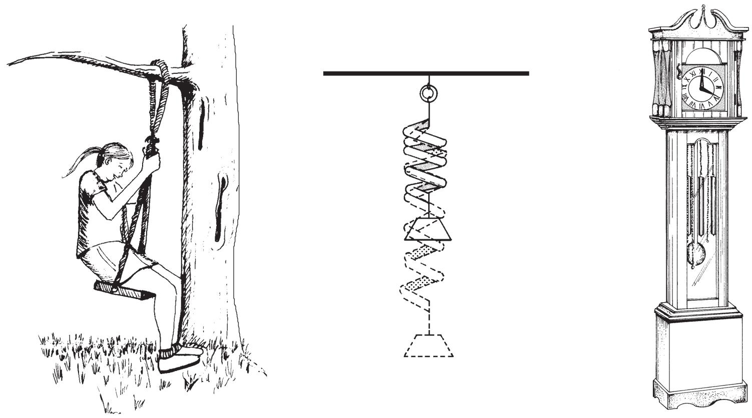


Figure 8.16
All oscillating objects have natural frequencies.

Resonance

The Tacoma Narrows bridge disaster is a historic example of what can happen when an outside force sets up a motion that matches the natural frequency of an object. On November 7, 1940, an intense wind caused the bridge to begin *twisting* along its entire length. The back-and-forth twisting motion matched the natural twisting frequency of the suspended bridge. As the blowing wind gave more and more energy to the torsional motion of the bridge, the bridge twisted back and forth with larger and larger amplitude. Finally, at approximately 11:00 A.M. the bridge tore itself apart.

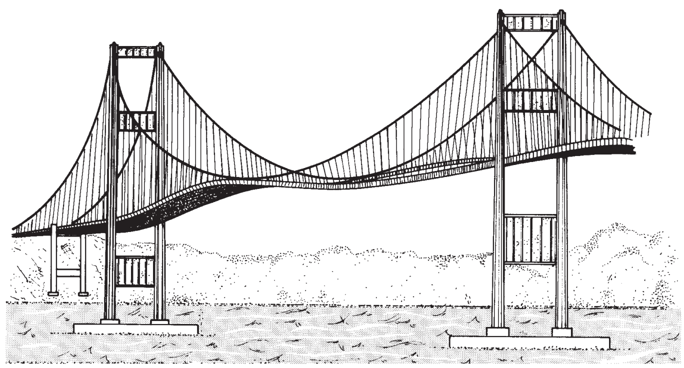


Figure 8.17
Tacoma Narrows bridge undergoing a twisting motion

A force can be applied to any object like the Tacoma Narrows bridge at regular intervals. The object is forced to oscillate at the frequency of the applied force. The motion of the object is called **forced vibration**. When the frequency of a forced vibration matches the natural frequency of an object, the object vibrates with increasing amplitude. This condition is called **resonance**. Resonance happens because the source of forced vibrations transfers energy to the object as work—a force is applied to the object in the direction of its motion during each oscillation.

You can demonstrate resonance with a swing. A swing is a pendulum with a natural frequency of oscillation. You "pump" a swing by leaning back and pushing your legs forward at the same point in each oscillation. You can also push someone else in the swing by applying forces repeatedly at just the right time. In either case, you are the source of forced vibration. If the frequency at which you apply forces equals the natural frequency of the swing, the swing's amplitude increases. This is resonance.

On the other hand, when the frequency of a forced vibration does not match the natural frequency of an object or mechanical system, the object's amplitude does not increase substantially. You can prove this by trying to "pump" a swing by pushing it at a frequency of forced vibration that's different from the natural frequency of the swing. You'll find that you can't build up much amplitude in the swing. That's because your successive *pushes* are *out of phase* with the natural frequency of the swing.

Resonance is very important in mechanical systems. Engineers, architects, and technicians purposely match forced oscillations and natural frequency when resonance is desired. Alternatively, they work to avoid or dampen undesired resonance. (Shock absorbers are used to dampen oscillation. These devices convert the kinetic energy of the oscillation into internal energy of a fluid inside the shock absorber.) Vibrations are almost always produced in machines when they operate. Structures also vibrate when forced vibrations are present. Vibrating machines, earthquakes, passing trucks, and sonic booms are a few examples of forced vibrations that can affect structures and machines. These vibrations don't usually represent a major problem—unless they coincide with a natural frequency of the machine or structure.

When they do coincide, however, resonance occurs. Then the amplitude of the vibration increases dramatically. The result is excessive wear and shortened life expectancy of the machine or structure or—in the case of an earthquake—instant damage. Vibration-isolation pads are used to isolate vibration caused by heavy, high-speed machines. Special shock absorbers are built into the structures of tall buildings to damp out vibrations (swaying) caused by wind forces.

Vibrations produced in internal-combustion engines are controlled to avoid resonance. During design, engines can have cylinders added, subtracted, or repositioned. The V-angle in a V-8 or V-6 can be adjusted. Or the firing order of the engine can be changed. Within the engine itself, a flywheel is added on the rear of the crankshaft and a vibration damper (harmonic balancer) is added to the front. Both help reduce and damp out engine vibrations. Engine mounts made of springs or rubber bushings are designed to eliminate vibrations that might cause resonance in the frame or body of the car (see Figure 8.18).

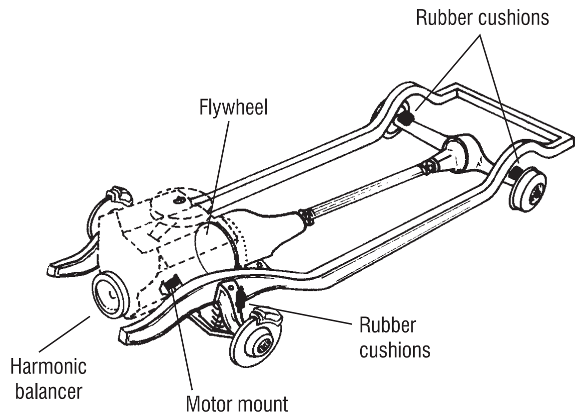


Figure 8.18
Vibration-reducing devices in an automobile

Resonance of a nonmechanical form is important in electrical systems. When a capacitor and an inductor are connected in parallel across an AC source, as shown in Figure 8.19, they form an *LC circuit*. This is also called a *resonant circuit*. Electrical energy flows back and forth (oscillates) between the inductor and the capacitor. It does so at a rate that depends on the natural frequency of oscillation of the LC circuit. Resonant LC circuits are used in many types of electronic equipment, such as radios, amplifiers, tuners, and television sets. If you take a course in electronics, you'll learn a lot more about resonant circuits.

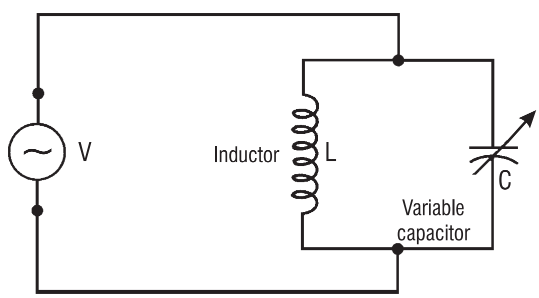


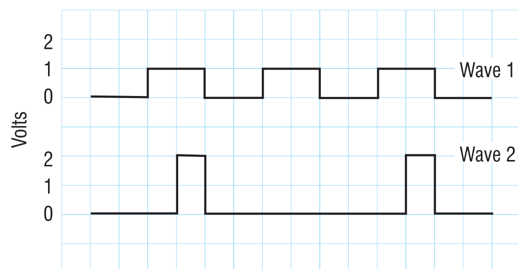
Figure 8.19
Resonant LC circuit in an electrical system

Summary

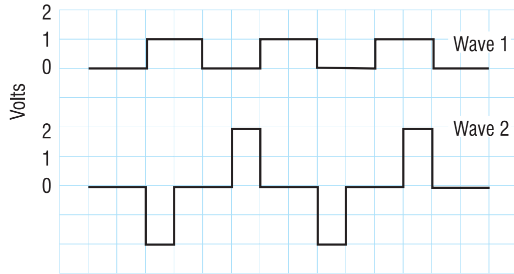
- When two or more waves in a medium pass through the same place at the same time, the waves overlap and interfere with each other.
- When two or more waves in a medium interfere, the overall wave displacement is equal to the sum of the displacements caused by the individual waves. This is called the principle of superposition.
- When two waves of equal frequency are in phase and interfere, the resultant wave has the same frequency but the amplitude is the sum of the individual amplitudes. This is called constructive interference.
- When two waves of equal frequency are out of phase and interfere, the resultant wave has the same frequency but the amplitude is the difference between the individual amplitudes. This is destructive interference.
- The natural frequency of an object or system is that frequency at which the object will vibrate on its own when set into motion.
- Resonance between a source of vibration and another object or system exists when the frequency of the vibrating source matches the natural frequency of the object. In resonance, the amplitude of vibration increases rapidly.

Exercises

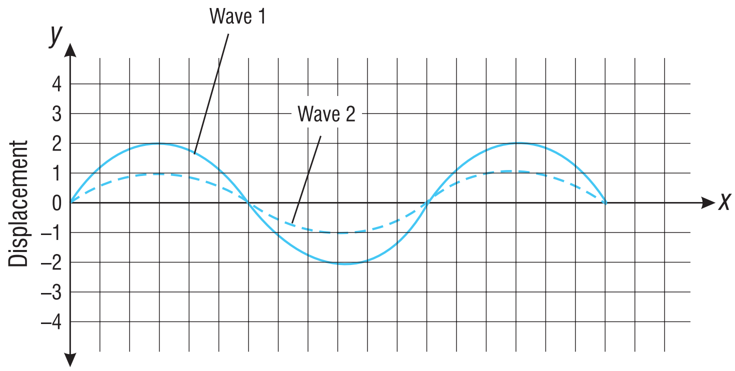
1. What is the general term used to describe what happens when two or more waves of the same type overlap in the same medium?
2. When two waves of equal frequency are in phase and interfere, they produce a resultant wave whose amplitude is larger than either wave. This is called ____________.
3. When two waves of equal frequency are out of phase and interfere, they produce a resultant wave whose amplitude is smaller than either wave. This is called ____________.
4. The two voltage signals at the right are sent through a wire simultaneously. Sketch the resulting wave.



5. The two voltage signals at the right are sent through a wire simultaneously. Sketch the resulting wave.



6. Trace or redraw waves 1 and 2 onto your paper. Sketch the wave resulting from the interference of 1 and 2.



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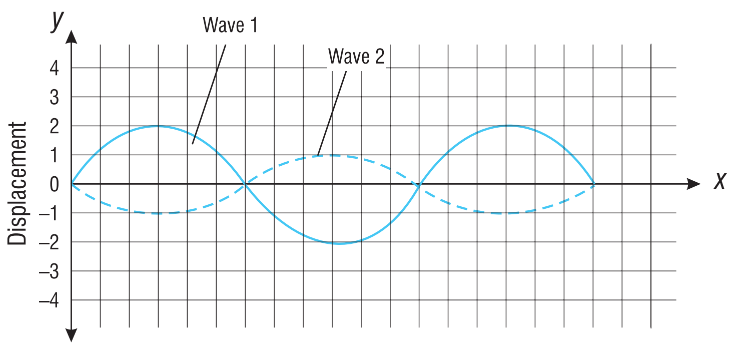
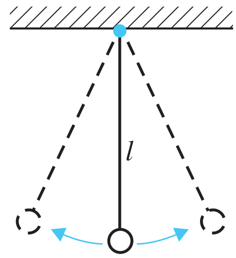


8. What is the value of displacement at the nodes of a standing wave in a stretched string?

9. When two interfering sound waves are very close in frequency, the resulting combined wave has regular, periodic variations in loudness. What are these variations called?

10. A pendulum consists of a mass, called the bob, suspended by a string. When the bob is pulled to one side and released, it swings back and forth. The period of a pendulum is the time required for the bob to complete one back-and-forth oscillation. Describe how you could measure the natural frequency of oscillation of a pendulum.



11. As long as the maximum angle of displacement of a pendulum is less than about 15°, the pendulum's period T depends on only its length l. The period is given by the following formula, where g is the acceleration of gravity.

$$T = 2\pi\sqrt{\frac{l}{g}}$$

(a) What is the frequency of a pendulum that is 1.2 m long?

(b) The Foucault pendulum in the National Museum of American History has a period of 8.0 s. How long is this pendulum?

12. The strings on a guitar vibrate to cause sound. Each string is anchored to the body of the guitar at the bridge. What role does the bridge of the guitar play in producing a standing wave in a guitar string?

13. The pilot of a twin-engine airplane hears a loud and annoying beat caused by the engines. This means that the engines are turning at slightly different rates. To eliminate the beat, the pilot "throttles down," or slowly decreases, one engine's rpm. If the beat frequency increases, are the engines' frequencies getting closer together or farther apart?