8.1 Properties of Waves

Objectives

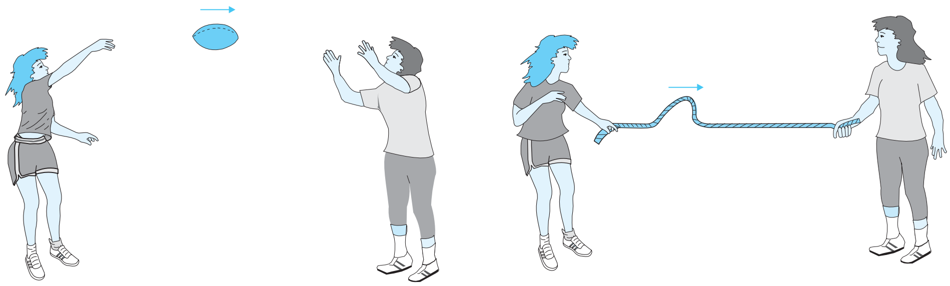
- Describe how a mechanical wave transfers energy through a medium.
- Explain the difference between a transverse wave and a longitudinal wave.
- Define amplitude and wavelength.
- Define frequency and period, and the relationship between them.
- Define wave speed.
- Solve problems using amplitude, wavelength, frequency, period, and speed.
- Describe the properties of sound.

INTERNET connection

To find out more about properties of waves, follow the links at **www.learningincontext.com.**

In previous chapters you have studied various ways of transferring energy and converting it from one form to another. Driving a nail with a hammer, using high-pressure steam to turn a turbine, and using a battery to force current to flow in a circuit all involve transfers of energy. Each of these also involves a transfer of matter.

Wave motion is another way to transfer energy, but without a transfer of matter. Figure 8.1 illustrates the difference. In 8.1a, energy is transferred when someone throws a football. Matter is also transferred—the ball carries energy and mass from the person on the left to the person on the right.



(a) A thrown football carries energy from one person to another.

(b) A wave pulse in a rope carries energy from one person to another.

Figure 8.1
Two ways of transferring energy:
(a) by transferring matter, and (b) by using a wave pulse

In Figure 8.1b, the same two people hold the ends of a rope. The person on the left gives the rope a sudden up-and-down shake. This single disturbance of the rope is called a *wave pulse*. The pulse carries energy down the rope to the person on the right. In this case, energy is transferred from one person to another but no mass is transferred between the two.

Mechanical Waves

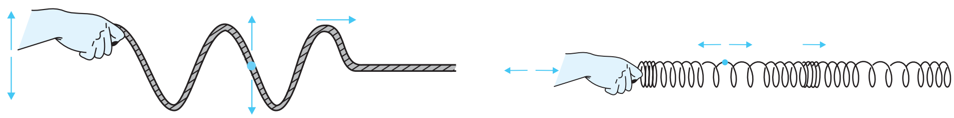
The wave pulse in Figure 8.1b is an example of a **mechanical wave**. A mechanical wave requires a medium—such as a rope, air, water, or soil—to transfer the energy of the wave from one place to another. The wave motion begins at a source. The source causes molecules in the medium to move up and down, right and left, or in and out around their undisturbed (equilibrium) positions. Because of the electrical forces between molecules, vibrating molecules cause adjacent molecules to vibrate also. A wave is formed as the vibration is passed along, from molecule to molecule, in the medium. The molecules always return to their original, equilibrium positions—mass is not transferred as a wave passes through a medium.

In the next chapter you will learn about *electromagnetic waves*. These are not the same as mechanical waves. Electromagnetic waves also transfer energy, and they have many properties in common with mechanical waves, but they do not require media. They travel through a vacuum.

Transverse and Longitudinal Waves

A **wave pulse** is produced if the source makes a single disturbance that travels through the medium. If the source vibrates repeatedly, it produces a **periodic wave**. The person in Figure 8.1b can transfer energy with a periodic wave by moving the end of the rope repeatedly up and down (see Figure

8.2a.) Notice that the waves in the rope travel horizontally but the particles in the rope move vertically. These are called *transverse waves*. In a **transverse wave**, the particles in the medium move in a direction perpendicular to the direction of wave motion.



(a) A transverse wave. The wave moves to the right. Particles in the rope move up and down.

(b) A longitudinal wave. The wave moves to the right. Particles in the spring move right and left.

Figure 8.2
A transverse wave in a rope and a longitudinal wave in a Slinky. Both are periodic waves.

In a **longitudinal wave**, the particles in the medium move in the same direction as (or parallel to) the wave propagation. You can generate a longitudinal wave in a coiled spring Slinky, as shown in Figure 8.2b. The end of the spring can be pushed or pulled quickly one time to generate a wave pulse or repeatedly pushed and pulled to generate a periodic wave. In Figure 8.2b, the particles in the Slinky vibrate right and left and the wave propagates from left to right. Particles in the Slinky move parallel to the direction of the wave.

In periodic waves, such as those illustrated in Figure 8.2, the smallest section of the wave that repeats at regular intervals is called the *waveform*. Most periodic waves have *sinusoidal* waveforms because they have exactly the same appearance as the graph of a sine (or cosine) function. You will investigate these functions in the exercises.

Amplitude

Suppose one end of a long rope is attached to a distant wall and the rope is stretched tightly. (We choose a distant wall so we don't have to worry about reflected waves coming back from the wall. We will deal with reflections later.) The other end of the rope is moved up and down repeatedly in a direction perpendicular to the rope's length. (Are the waves produced transverse or longitudinal?) Figure 8.3a on the next page is a "snapshot" of the rope showing the outward-traveling wave. It is called a snapshot because it is what you would record on film with a high-speed camera. Figure 8.3b is a snapshot of the same rope when the displacement from equilibrium is less.

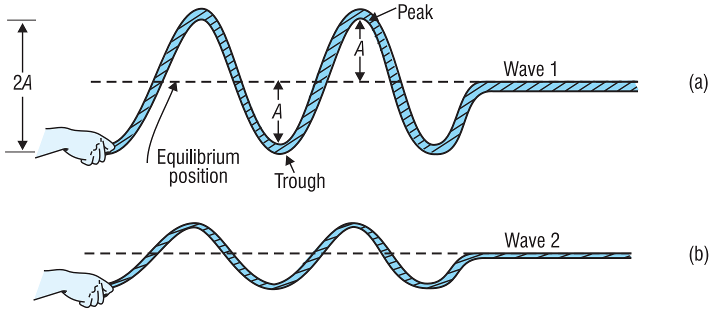


Figure 8.3
The amplitude *A* of a wave is the distance from the equilibrium, or average, position to the peak or trough.

Each of the highest points on the rope is called a *peak*, or *crest*. Each of the lowest points is called a *trough*. The distance from the rope's equilibrium position to the crest is called the **amplitude** of the wave. (The amplitude is also the distance from the equilibrium position to the trough.) A wave's amplitude usually is determined by the source of the wave. The larger the displacement of the source, the larger the amplitude. Which wave in Figure 8.3 has a larger amplitude? Notice that it takes more work (in a given medium) to generate a wave with a larger amplitude—the force at the wave's source must be moved through a longer distance. Therefore, the amplitude of a wave determines how much energy the wave transfers.

Wavelength

The length of the repeating pattern is another important characteristic of a periodic wave. This length is called the **wavelength**. We use the Greek letter *lambda* λ to represent wavelength. You can measure the wavelength from a snapshot of a wave by measuring the shortest distance between two points where the pattern repeats. For example, λ is shown in Figure 8.4 as measured from peak to peak, from trough to trough, and from equilibrium point to equilibrium point.

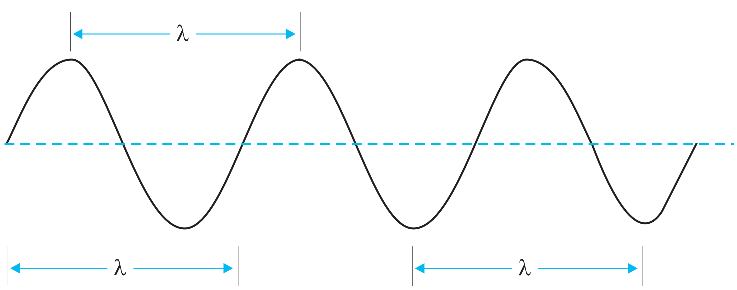
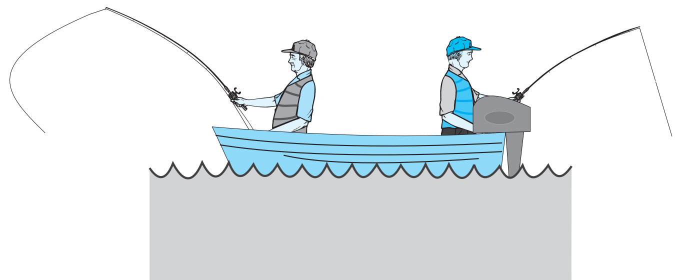


Figure 8.4
The wavelength can be measured between any two points where the wave pattern repeats.

Example 8.1 Amplitude and Wavelength of Water Waves

Water waves pass by a 16-ft fishing boat. There are exactly 12 crests from front to back of the boat. The vertical distance between a crest and a trough is 7 inches. What are the amplitude and wavelength of the water waves?

Solution:



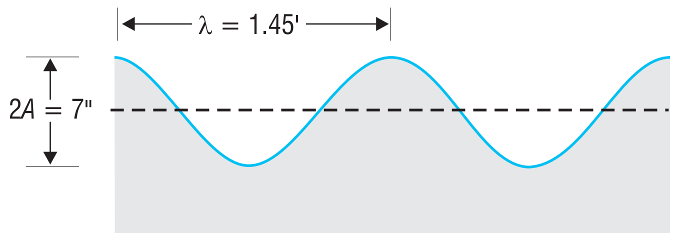
The wave amplitude is one-half the distance from crest to trough.

$$\text{Amplitude} = \tfrac{1}{2}\,(7\text{ in}) = 3.5\text{ in}$$

Since exactly 12 crests extend horizontally between the front and back of the boat, there are 11 crest-to-crest distances. The wavelength λ is one crest-to-crest distance.

$$11\lambda = 16\text{ ft}$$

$$\lambda = \frac{16\text{ ft}}{11} = 1.45\text{ ft}$$



The amplitude is 3.5 inches and the wavelength is 1.45 feet.

Frequency and Period

Frequency was defined in Section 3.3 as the rate of occurrence of events, or the number of events occurring per unit time. For waves, **frequency** is the number of complete waves (or cycles) per unit time that pass a point in the medium. Frequency is measured in hertz (Hz). One hertz is one cycle per second.

The **period** of a wave is the amount of time it takes for one complete wavelength to pass a point in the medium. The frequency f and period T of a wave are reciprocals:

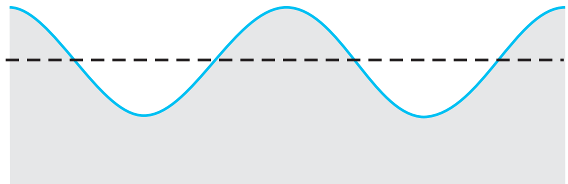
$$f = \frac{1}{T} \quad \text{and} \quad T = \frac{1}{f}$$

Frequency and period apply to periodic waves only, not wave pulses. They depend only on the source of the wave, not on the medium or the speed of the wave.

Example 8.2 Frequency and Period of Water Waves

Using a stopwatch for the water waves in Example 8.1, you determine that 3 wave crests pass the bow of the boat in 2.6 seconds. What are the frequency and period of the water waves?

Solution:



There are 2 wave cycles between 3 crests.

$$f = \frac{\text{number of cycles}}{\text{time interval}}$$

$$= \frac{2 \text{ cycles}}{2.6 \text{ s}}$$

$$= 0.77 \text{ cycle/s} \quad \text{or} \quad 0.77 \text{ Hz}$$

The wave period is the reciprocal of the frequency.

$$T = \frac{1}{f}$$

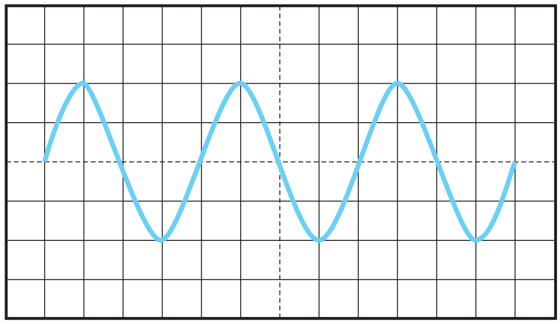
$$= \frac{1}{0.77 \text{ cycle/s}}$$

$$= 1.3 \text{ s/cycle}$$

By convention, the unit "cycle" is dropped when reporting the period of a wave. The wave frequency is 0.77 hertz and the period is 1.3 seconds.

Example 8.3 Frequency and Period of a Sine Wave Trace

A voltage signal is displayed on an oscilloscope screen as shown in the illustration below. The display is called a *trace*. The trace is a sine wave. The vertical scale is the voltage, and the horizontal scale is time. The *time/division* setting on the oscilloscope is 0.05 s/div. What are the frequency and period of the signal?



Solution: The trace shows that 3 complete wavelengths, or cycles, span 12 time divisions. Each time division is 0.05 second. Thus, the time interval is the product:

$$\text{Time interval} = (12 \text{ div})(0.05 \text{ s/div}) = 0.6 \text{ s}$$

$$f = \frac{\text{number of cycles}}{\text{time interval}} = \frac{3 \text{ cycles}}{0.6 \text{ s}} = 5.0 \text{ cycles/s}$$

$$= 5.0 \text{ Hz}$$

The wave period is the reciprocal of the frequency.

$$T = \frac{1}{f} = \frac{1}{5.0 \text{ cycle/s}} = 0.2 \text{ s/cycle}$$

The signal frequency is 5.0 hertz and the period is 0.2 second.

Speed

You can measure the speed of a wave pulse or periodic wave the same way you measure the speed of a moving object. If a point on the wave (for example, a peak or a trough) moves a distance Δx in a time interval Δt, the wave speed is $v = \Delta x/\Delta t$. Note that v is the speed of a point traveling with the wave, not the speed of the medium.

The wave travels a distance of one wavelength in a time interval of one period. Therefore, the speed v of a wave can be calculated if you know the wavelength λ and period T (or frequency f).

$$v = \frac{\text{distance traveled}}{\text{time interval}} = \frac{\lambda}{T}$$

Since $T = \frac{1}{f}$,

$$v = \lambda f$$

The speed of a mechanical wave depends on the medium through which the wave travels. For example, sound waves travel faster through denser media. Sound travels through air (at 20°C) at 343 m/s, through water (at 25°C) at 1490 m/s, and through steel at 5700 m/s. As a comparison, the speed of an electromagnetic wave (a nonmechanical wave) in vacuum is 3×10^8 m/s.

Example 8.4 Speed of Water Waves

What is the speed of the water waves in Examples 8.1 and 8.2?

Solution: From Example 8.1, the wavelength λ is 1.45 feet. From Example 8.2, the frequency f is 5.0 Hz.

$$\begin{aligned} v &= \lambda f \\ &= (1.45\text{ ft})(5.0\text{ cycles/s}) \qquad \text{[Drop “cycles”]} \\ &= 7.25\text{ ft/s} \end{aligned}$$

The speed of the water waves is 7.25 feet per second.

Sound

Refer to Appendix F for a career link to this concept.

Sound is a longitudinal mechanical wave. The source of a sound wave vibrates rapidly, causing *pressure variations* in the medium. A guitar string, a drumhead, and your vocal cords are sources of sound waves. Their vibrations cause sound to propagate through air.

Figure 8.5 on the next page illustrates how a portion of a vibrating membrane, such as a stereo speaker, causes sound. When the membrane moves forward, air molecules next to it are pushed forward. This creates a region of increased air density and pressure. This region is called a *compression*. When the membrane moves backward, the air in front of it expands, decreasing the air density and pressure. This region is called a *rarefaction*.

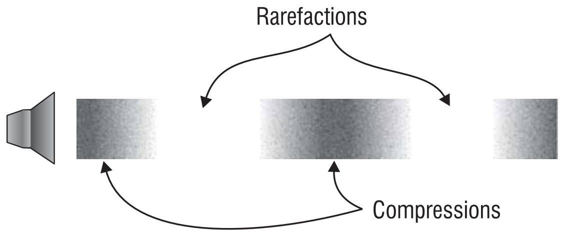


Figure 8.5
A sound wave is a pattern of compressions and rarefactions.

If the membrane vibrates regularly, a pattern of pressure variation is created. The pattern is a sound wave. The sound wave is shown in Figure 8.6 as a series of snapshots and graphs for three instants in time. Each graph shows how the air pressure varies as a function of distance in front of the source. This kind of graph shows the *spatial variation* of air pressure at one instant in time caused by the sound wave. Each graph is a sinusoidal waveform.

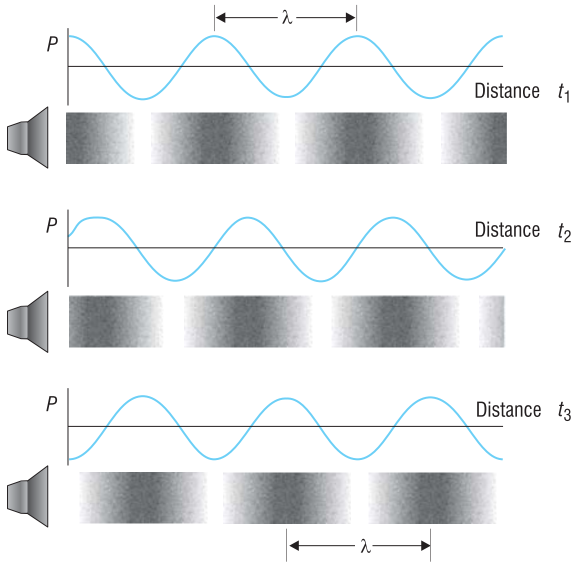


Figure 8.6
Sound is a longitudinal wave consisting of compressions and rarefactions in the air.

You also can measure air pressure at a given location over a time interval. Figure 8.7 shows a graph of the measurement. This graph is also a sinusoidal, showing the *time*, or *temporal variation*, of air pressure at one location caused by the sound wave.

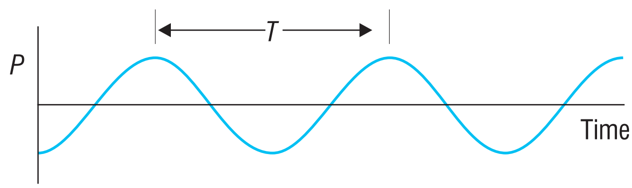


Figure 8.7
The time variation of pressure at one location caused by a sound wave. The pressure-versus-time plot is sinusoidal.

You can measure the wavelength of the sound (or pressure) wave from the spatial variation. The wavelength is the distance required to complete one wave cycle. You can measure the period from the temporal variation. The period is the time required to complete one wave cycle. The amplitude can be measured from either graph.

When you hear sound, energy is transferred from a source to your ears. The pressure variations in the sound wave cause your eardrum to vibrate at the same frequency as the wave. Your brain interprets the vibrations detected by your ears. The loudness of the sound depends on the amplitude of the pressure wave. The human ear is sensitive to extremely small amplitudes—less than one billionth of normal atmospheric pressure. But our ears are not sensitive to a large frequency range. Most people can hear sounds between approximately 20 Hz and 16,000 Hz. We lose sensitivity, especially to higher frequencies, with long-term exposure to loud sounds and as we age.

Example 8.5 The Wavelength of Middle C

On a piano, middle C has a frequency of 262 Hz. If the speed of sound in air is 343 m/s, what is the wavelength of the sound wave produced by the piano's middle C?

Solution: Solve the wave-speed equation for λ:

$$v = \lambda f$$

$$\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{262 \text{ cycles/s}} = 1.31 \text{ m}$$

The wavelength is 1.31 meters.

Summary

- In a transverse mechanical wave, particles oscillate in a direction perpendicular to the direction of the wave motion.
- In a longitudinal wave, particles oscillate parallel to the direction of wave motion.
- The amplitude of a mechanical wave is the maximum displacement of particles in the medium from their equilibrium positions.

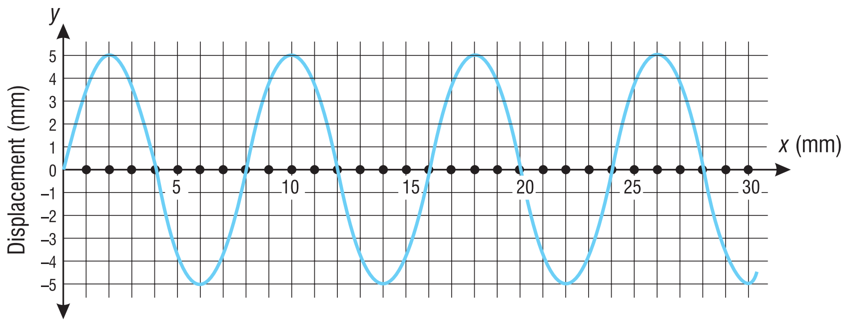
- The wavelength is the shortest distance between two points on the wave where the pattern repeats.
- Frequency is the number of complete cycles or oscillations per unit time.
- Period is the time required for one complete cycle, or oscillation. Period is the reciprocal of frequency.
- The speed of a wave is how fast the wave travels along the direction of propagation. The wave speed equals the product of its wavelength and its frequency. $v = \lambda f$
- Sound is a longitudinal wave that propagates through a medium as pressure variations.

Exercises

1. What are two ways of transferring energy from one place to another?
2. A ________________ (mechanical or electromagnetic) wave requires a medium containing matter for propagation. A ________________ (mechanical or electromagnetic) wave does not require a medium.
3. What is the primary difference between a transverse wave and a longitudinal wave?
4. In a periodic wave traveling down a stretched string, the vertical distance between a crest and a trough is 1.0 cm. What is the amplitude of the wave?
5. Match the following definitions to the words at the right.

___	The distance between two adjacent crests on a transverse wave	a. frequency
___	The distance from the peak of a transverse wave to the equilibrium, or average, position of the wave	b. period
___	The distance between two adjacent high-pressure regions in a sound wave	c. wavelength
___	The time one cycle of a wave takes to pass by a fixed location	d. amplitude
___	The rate at which complete cycles in a wave pass by a fixed location	e. wave speed
___	The product of wavelength and frequency	

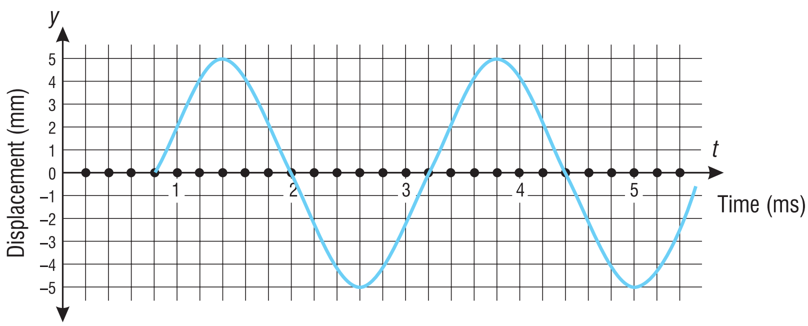
6. A snapshot of a transverse wave traveling down a stretched string is shown below. The horizontal axis shows the distance along the string, in millimeters. The vertical axis shows the displacement of the string, in millimeters.



(a) What is the amplitude of the wave?

(b) What is the wavelength?

7. The graph below shows the displacement (in millimeters) as a function of time (in milliseconds) for a point on the string of Exercise 6.

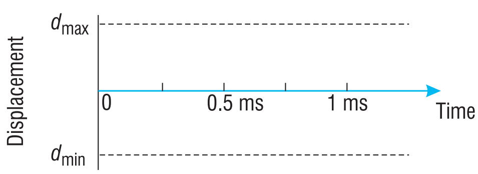


(a) What is the period of the wave?

(b) What is the frequency?

(c) What is the wave speed?

8. When you hear sound, the pressure wave moves your eardrum back and forth between maximum and minimum displacements d_{max} and d_{min}. Sketch a graph of your eardrum's displacement in response to each of the following single-frequency tones. Use axes similar to those below.



(a) A 1-kHz (1000 Hz) tone. Show one complete cycle.

(b) A 2-kHz tone. Show two complete cycles.

9. While waiting to catch a wave, a surfer estimates that 20 meters separate the crests of adjacent waves. A wave passes the surfer every 8 seconds. What is the approximate speed of the water waves?

10. The wavelength of blue light is 470 nm. (1 nm = 10^{-9} m) The speed of light is 3×10^8 m/s.

    (a) Find the frequency of blue light.

    (b) Find the period.

11. What property of a sound wave (or pressure wave) changes when the loudness, or intensity, of the sound increases?

12. The speed of sound in steel is 5700 m/s. How far apart are the compressions caused by a sound wave that has a frequency of 8400 Hz?

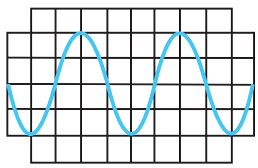
13. A bat emits short pulses of high-frequency sound waves (called ultrasound). By listening to echoes of the emitted ultrasound, the bat can detect and catch flying insects. What is the frequency of the sound emitted by the bat if its wavelength is 3.2 mm? The speed of sound in air is 343 m/s.

14. A radio station broadcasts an FM electromagnetic wave at a frequency of 92.5 MHz. The speed of electromagnetic waves is 3×10^8 m/s.

    (a) What is the period of the wave in seconds?

    (b) What is the wavelength in meters?

15. The oscilloscope trace below is for an alternating voltage signal. The volts/division (vertical scale) control is set at 2 volts/div. The time/division (horizontal scale) control is set at 0.2 msec/div.



    (a) What is the amplitude of the voltage signal?

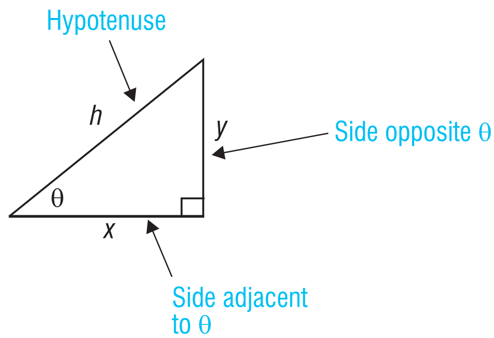
    (b) What is the period?

    (c) What is the frequency?

16. A right triangle contains one 90° angle. The side opposite the right angle is the hypotenuse and the other two sides are called the legs. The hypotenuse is the longest side of a right triangle.

Let h represent the length of the hypotenuse, and let x and y represent the lengths of the legs, as shown at the right. h is related to x and y by the Pythagorean theorem:

$$h^2 = x^2 + y^2$$



The *sine*, *cosine*, and *tangent* of an angle in a right triangle are defined with ratios involving the lengths of the sides of the triangle. These ratios are abbreviated *sin*, *cos*, and *tan*. For the angle θ shown above, y is the length of the opposite side and x is the length of the adjacent side.

$$\sin\ \theta = \frac{y}{h} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\ \theta = \frac{x}{h} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\ \theta = \frac{y}{x} = \frac{\text{opposite}}{\text{adjacent}}$$

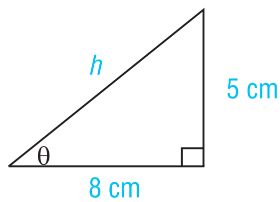
For the triangle at the right, calculate:

(a) The length of the hypotenuse.

(b) sin θ

(c) cos θ

(d) tan θ



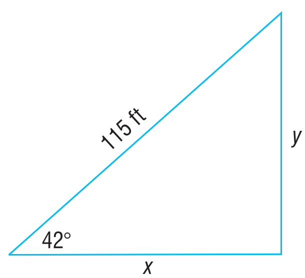
17. You can use your calculator to find values for the sine, cosine, and tangent of an angle. Most calculators will work with either degrees or radians. We will work with degrees, so make sure your calculator is in **DEG** mode. To find the sine of 60°, press **60**, then **sin**. Your calculator should read 0.866025403. Rounded to three significant figures, sin 60° = 0.866.

For the triangle at the right, calculate:

(a) sin 42°, cos 42°, and tan 42°.

(b) y. (Use the ratio for sin θ given in Exercise 16.)

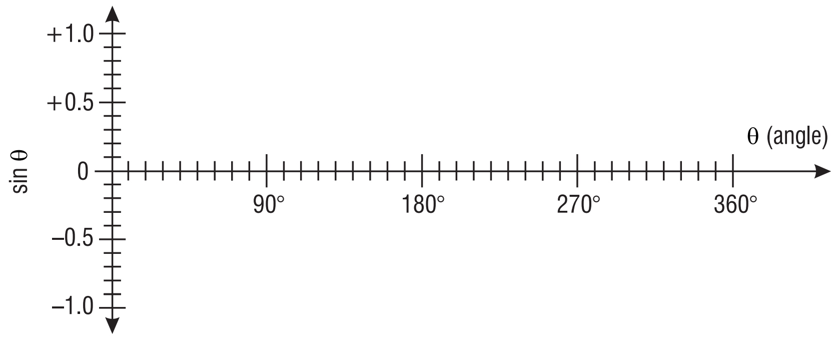
(c) x. (Use the ratio for cos θ given in Exercise 16.)



18. (a) Make a table of values for sin θ, for values of θ from 0° to 360°, similar to the one below. Use increments of 30°, and round your values to two significant figures.

θ	sin θ	θ	sin θ	θ	sin θ
0°		120°		240°	
30°		150°		270°	
60°		180°		300°	
90°		210°		330°	
				360°	

(b) Plot the data from your table, using coordinate axes as shown below and graph paper. The result is a sine wave. How many cycles of a sine wave are between 0° and 360°?



(c) How would you change the horizontal axis of your graph to show the sine wave with units of radians instead of degrees?

19. A *wave function* can be written in the form

$$y = A \sin \omega t$$

where A is the amplitude of the wave, ω is the angular frequency, and t is time. In this formula, the angle θ is the product ωt. This is called the *phase* of the wave. In a wave function, the phase has units of radians, not degrees.

Let y represent the time-varying voltage output of a signal generator. The maximum value of y is 5 volts, and the minimum value is –5 volts. The signal has a frequency of 10 Hz. (Remember, $\omega = 2\pi f$.)

Plot the wave function, with y on the vertical axis and t on the horizontal axis. Show one cycle of the wave, from $\omega t = 0$ (or $t = 0$ s) to $\omega t = 2\pi$ (or $t = 2\pi/\omega$). Compare your plot to the one from Exercise 18. How are they alike? How are they different?