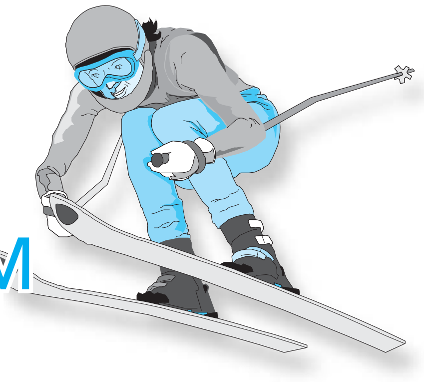
7.1 LINEAR MOMENTUM



Objectives

- Define linear momentum.
- Explain the relationship between force and rate of change of momentum.
- Define impulse.
- Explain the relationship between impulse and change in momentum.
- Explain Newton's third law of motion.
- Explain the law of conservation of momentum.
- Use linear momentum, impulse, and conservation of momentum to solve problems.

INTERNET *connection*

To find out more about linear momentum, follow the links at **www.learningincontext.com.**

You learned in Chapter 1 that inertia is the tendency of an object to resist changes in motion. Newton's first law of motion describes inertia:

> *An object at rest will remain at rest, and an object in motion will continue to move in a straight line, unless the object is acted on by a net force.*

An object's mass tells you how much inertia it has.

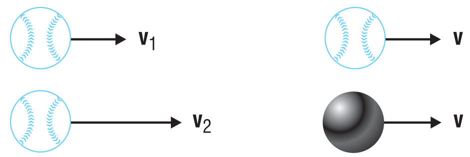
Objects in motion have *momentum* also. An object's momentum tells you how difficult it is to stop the object. Some problems involving motion are easier to solve using momentum instead of work, energy, and acceleration.

Refer to Appendix F for a career link to this concept.

Momentum

Have you ever caught a softball that was tossed gently and then caught the same softball thrown at high speed? For each catch, the ball has the same *mass*, but the *speeds* are different. The fast-moving softball is harder to stop—it has more momentum (see Figure 7.1a).

Now compare catching a softball to catching a heavy iron shot, when both objects are tossed at the same *speed*. (An iron shot is used in shot-put competitions.) The *masses* are different, and the more massive shot is harder to stop—it has more momentum (see Figure 7.1b).



(a) The faster-moving softball has more momentum.

(b) The shot has more mass and more momentum.

Figure 7.1
Objects with higher speeds and higher mass have more momentum.

So, to compare the momentum of two softballs, or a softball and an iron shot, you need to consider both the mass and the velocity. The momentum of an object increases as the mass increases and as the speed increases. For this reason, **linear momentum** is defined as the product of an object's mass m and its velocity $\mathbf{v}$. We use the symbol $\mathbf{p}$ to represent linear momentum.

$$\text{Linear momentum} = \text{mass} \times \text{velocity}$$

$$\mathbf{p} = m\mathbf{v}$$

Linear momentum is a vector quantity. The direction of $\mathbf{p}$ is the same as the direction of the velocity $\mathbf{v}$. In SI, the units of linear momentum are kg·m/s. The English units of linear momentum are slug·ft/s.

We call the product $m\mathbf{v}$ *linear* momentum because it is directed along a straight *line*. The direction of an object's linear momentum is the same as the direction of the object's velocity. In the next section you will learn about *angular* momentum—a property of an object moving in rotation. There is usually no question as to which momentum is involved in a situation. For this reason, *linear momentum* is usually referred to simply as *momentum*.

Example 7.1 Momenta of a Softball and an Iron Shot

A 9.5-ounce softball has a mass of 0.018 slug. A 16-pound shot has a mass of 0.50 slug. Suppose a softball is thrown at a speed of 60 ft/s. At what speed must a shot be tossed to have the same momentum as the softball?

Solution: The momentum of the shot equals the momentum of the softball.

$$p_{\text{shot}} = p_{\text{softball}}$$

$$m_{\text{shot}}\ v_{\text{shot}} = m_{\text{softball}}\ v_{\text{softball}}$$

$$v_{\text{shot}} = \frac{(0.018 \text{ slug})(60 \text{ ft/s})}{0.50 \text{ slug}}$$

$$= 2.16 \text{ ft/s}$$

To have the same momentum as the softball, the shot must be tossed at a speed of 2.16 feet per second.

Momentum and Newton's Second Law

Consider what happens when a bat strikes a softball (see Figure 7.2). The bat exerts a force on the ball and changes its direction and speed. In other words, the force accelerates the softball.

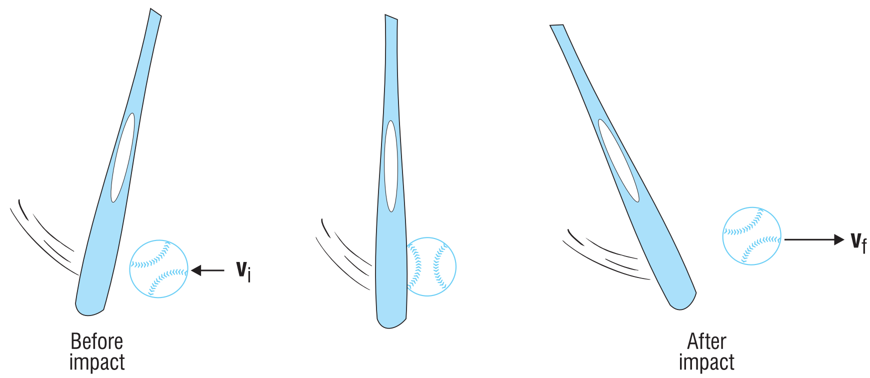


Figure 7.2
When a bat strikes a softball, it changes the momentum of the ball.

In Chapter 4 you learned the relationship between the net force exerted on an object, the mass of the object, and the object's acceleration. This relationship is expressed in Newton's second law.

$$\mathbf{F} = m\mathbf{a}$$

The acceleration of the softball is its rate of change of velocity.

If the velocity changes by an amount $\Delta \mathbf{v}$ in a time interval Δt, the acceleration is the ratio of $\Delta \mathbf{v}$ to Δt.

$$\mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t}$$

Substitute this expression for **a** into the equation for Newton's second law.

$$\mathbf{F} = m\frac{\Delta \mathbf{v}}{\Delta t} = \frac{m\Delta \mathbf{v}}{\Delta t}$$

Let $\mathbf{v}_i$ represent the softball's initial velocity and $\mathbf{v}_f$ represent the final velocity. Then you can write the numerator of the equation above as follows:

$$m\Delta \mathbf{v} = m(\mathbf{v}_f - \mathbf{v}_i) = m\mathbf{v}_f - m\mathbf{v}_i$$

$$m\Delta \mathbf{v} = \mathbf{p}_f - \mathbf{p}_i$$

$$m\Delta \mathbf{v} = \Delta \mathbf{p}$$

In this equation, the ball's initial momentum is $\mathbf{p}_i$ ($= m\mathbf{v}_i$) and the final momentum is $\mathbf{p}_f$ ($= m\mathbf{v}_f$).

Therefore, the equation for Newton's second law can be written as the following ratio:

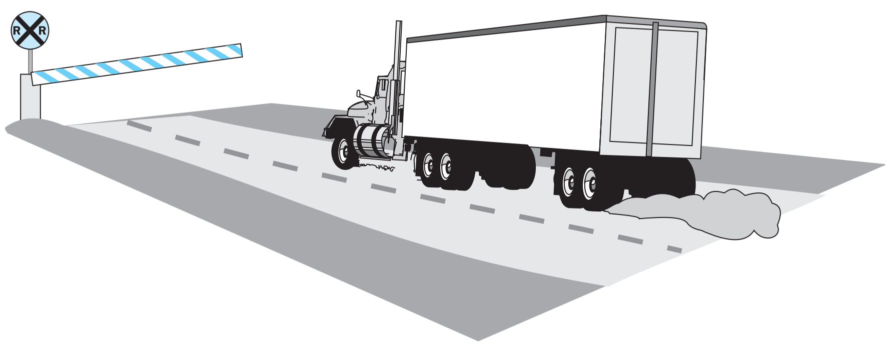
$$\mathbf{F} = \frac{\Delta \mathbf{p}}{\Delta t}$$

This equation states an alternative, equivalent form of Newton's second law.

> *A net force exerted on an object equals the rate of change of the object's linear momentum.*

Example 7.2 Braking Force and Momentum Change

A loaded tractor-trailer has a mass of 21,000 kg. If it is moving at a speed of 90 km/h, what braking force is required to stop the tractor-trailer in 30 seconds?



Solution: Let **F** represent the braking force, and let $\mathbf{v}_i$ and $\mathbf{v}_f$ represent the tractor-trailer's initial and final velocities. Since motion is in one dimension, solve the equations using magnitudes of the vector quantities.

$$\Delta p = p_f - p_i = mv_f - mv_i$$

Since the tractor-trailer stops, $v_f = 0$. Convert v_i to meters per second.

$$v_i = \left(90\frac{\text{km}}{\text{h}}\right)\left(1000\frac{\text{m}}{\text{km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 25\text{ m/s}$$

$$\Delta p = 0 - (21{,}000\text{ kg})(25\text{ m/s}) = -5.25\times 10^5\text{ kg}\cdot\text{m/s}$$

$$F = \frac{\Delta p}{\Delta t} = \frac{-5.25\times 10^5\text{ kg}\cdot\text{m/s}}{30\text{ s}}$$

$$= -1.75\times 10^4\text{ N} \qquad \left[\text{kg}\cdot\text{m/s}^2 = \text{N}\right]$$

The force is negative because it acts opposite the direction of the forward motion. The required braking force is 17,500 newtons.

Impulse

To change the speed and momentum of an object, a force must be applied for a period of time. For example, to accelerate a car from rest, the engine and drivetrain apply a force **F** for a time interval Δt. The greater the force and the longer it is applied, the greater the car's final speed and momentum. This idea leads to a quantity called *impulse*. **Impulse** is the product of force **F** and the time interval Δt over which the force acts.

$$\text{Impulse} = \mathbf{F}\Delta t$$

Impulse is a vector quantity. Its direction is the same as the direction of the force.

The impulse applied to a car determines the car's momentum change. To see why, write Newton's second law as rate of change of momentum and multiply both sides by Δt:

$$\mathbf{F} = \frac{\Delta\mathbf{p}}{\Delta t}$$

$$\mathbf{F}\Delta t = \frac{\Delta\mathbf{p}}{\Delta t}\Delta t$$

$$\mathbf{F}\Delta t = \Delta\mathbf{p}$$

Or in words,

$$\text{Impulse} = \text{momentum change}$$

Impulse is force times time. Therefore, the SI units of impulse are newton·seconds (N·s). These are equivalent to momentum units (kg·m/s). (Can you prove this?) But impulse units are written as N·s to distinguish impulse from momentum. The English units of impulse and momentum are also equivalent (see Table 7.1).

Table 7.1
Units of Force, Momentum, and Impulse

	SI Units	English Units
Force	N	lb
Momentum	kg·m/s	slug·ft/s
Impulse	N·s	lb·s

Impulse explains the importance of "follow-through" in many sports. A baseball or softball batter, a tennis player, or a golfer strikes a ball with a large force, but follows through with the swing to extend the time of contact of the force on the ball. A large force times a large time interval is a greater impulse, which results in a greater momentum change of the ball.

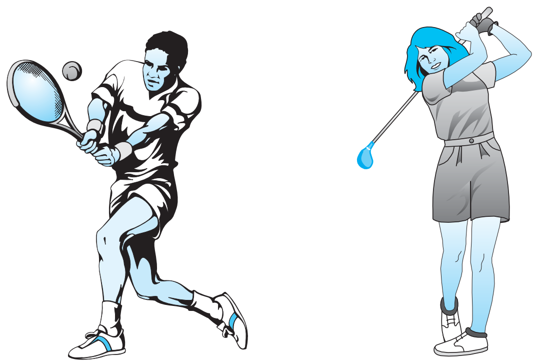


Figure 7.3
The change in momentum can be increased by increasing the force and/or the time interval over which the force acts. $\Delta p = F\Delta t$.

When calculating impulse using the product $F\Delta t$, we assume that F is constant. But the force can vary during the time interval Δt in many applications. For example, a bat exerts a force on a softball that varies in time. The force starts at zero, reaches a maximum value, and then decreases back to zero. The time-varying force of a bat striking a softball is graphed in Figure 7.4. In this type of impulse problem, we use the *average value* of the force to calculate $F\Delta t$.

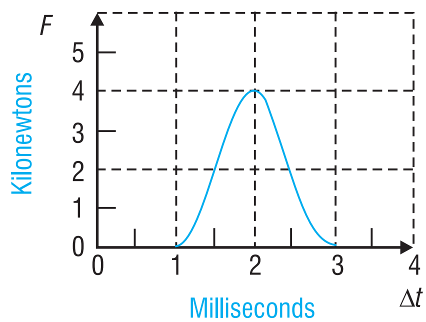


Figure 7.4
Force exerted by a bat on a softball as a function of time

Example 7.3 Impulse and Force on a Soccer Ball

In a penalty kick, a soccer player increases the speed of a ball from zero to 35 m/s. The mass of a soccer ball is 0.45 kg.

(a) What impulse does the soccer player give the ball?

(b) If the player's foot is in contact with the ball for 0.12 s, what is the average force exerted on the ball?

Solution: (a) The impulse equals the momentum change. The ball's initial speed is zero, so $v_i = 0$.

$$\text{Impulse} = \Delta p = p_f - p_i$$

$$\text{Impulse} = mv_f - mv_i = (0.45\text{ kg})(35\text{ m/s}) - 0$$

$$\text{Impulse} = 15.75\text{ N}\cdot\text{s}$$

(b) The impulse equals the average force times the time interval.

$$\text{Impulse} = F\Delta t$$

$$F = \frac{\text{Impulse}}{\Delta t} = \frac{15.75\text{ N}\cdot\text{s}}{0.12\text{ s}} = 131\text{ N}$$

The soccer player exerts an average force of 131 newtons on the ball.

Newton's Third Law of Motion

Refer to Appendix F for a career link to this concept.

In Example 7.3, we stated that the soccer player exerts a force on the ball. But isn't it also true that the ball exerts a force on the soccer player? A force is simply a push or a pull. But a force cannot be exerted on one object unless a second object exerts the force. Therefore, a force is an *interaction* between two objects. Newton stated this interaction in his **third law of motion:**

> *Whenever one body exerts a force on another body, the second body exerts an equal and opposite force on the first.*

Newton's third law says that forces always come in pairs, sometimes called *action* and *reaction* forces. Each force in a pair acts on a different object, and the two forces are equal in magnitude and opposite in direction. If the objects are labeled A and B, then $\mathbf{F}_{\text{A on B}} = -\mathbf{F}_{\text{B on A}}$.

For example, when a soccer player kicks a ball, his foot exerts a force on the ball. The ball exerts an equal and opposite force on the player's foot. When a flower vase is placed on a table, the vase exerts a force on the table (its weight). The table exerts an equal and opposite force on the vase. When fuel is burned in a rocket motor, the motor exerts a force on the expanding gas causing the gas to accelerate through a nozzle. The gas exerts an equal and opposite force on the rocket motor causing the rocket to accelerate.

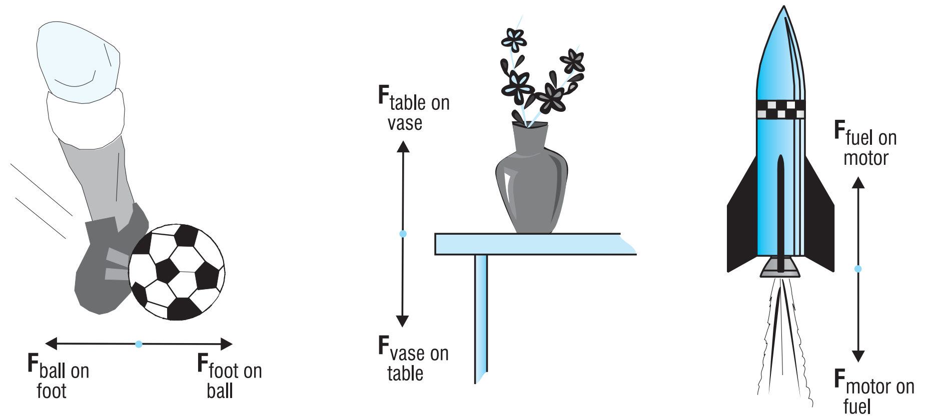


Figure 7.5
Forces always occur in pairs.

You can use Newton's laws and energy conservation to analyze the motion of objects such as those in Figure 7.5. But first you must define the "system." If the system is the soccer ball, a net force is acting on the system and it accelerates. But, suppose you enlarge the system to include the ball, the player, and the ground on which the player is standing. Now the only forces are action-reaction pairs, which are equal and opposite and cancel each other. There is no net force on this larger system. But you can still get important information about the motion of objects in this system if you use linear momentum.

Conservation of Linear Momentum

Consider the motion of two balls in a collision, as shown in Figure 7.6. The balls have different masses and initial velocities, and therefore different momenta before and after the collision. Let the system consist of both balls. In this case, the system does not lose or gain mass. This is called a **closed system**. The only forces involved in the closed system of Figure 7.6 are internal forces. A closed system on which no net external forces act is called an **isolated system**.

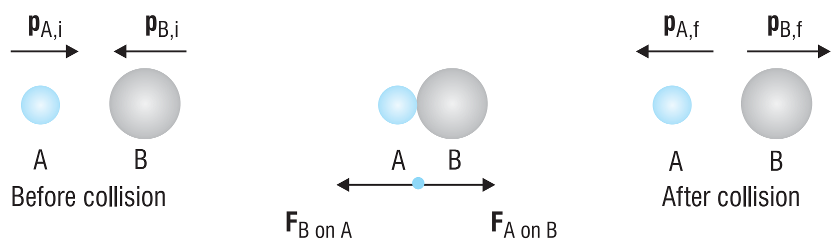


Figure 7.6
The momentum of each ball in a collision changes, but the total momentum of the system does not change.

Apply the impulse-momentum equation to the isolated system. Since the net external force is zero, the impulse applied to the system is zero and the system's momentum change is zero.

$$\mathbf{F}\Delta t = 0 = \Delta\mathbf{p}$$

If $\Delta\mathbf{p} = 0$, then $\mathbf{p}_f - \mathbf{p}_i = 0$, or $\mathbf{p}_f = \mathbf{p}_i$. The system's linear momentum does not change.

You can prove that the total momentum of the system in Figure 7.6 does not change as follows. Write the impulse-momentum equation for each ball.

$$\text{Ball A:}\quad \mathbf{F}_{\text{B on A}}\Delta t = \mathbf{p}_{\text{A,f}} - \mathbf{p}_{\text{A,i}}$$

$$\text{Ball B:}\quad \mathbf{F}_{\text{A on B}}\Delta t = \mathbf{p}_{\text{B,f}} - \mathbf{p}_{\text{B,i}}$$

The forces are action-reaction forces, internal to the system. From Newton's third law, they are equal and opposite. $\mathbf{F}_{\text{A on B}} = -\mathbf{F}_{\text{B on A}}$. The forces are applied over the same time interval Δt. Therefore, the impulses are equal and opposite.

$$\mathbf{F}_{\text{A on B}}\Delta t = -\mathbf{F}_{\text{B on A}}\Delta t$$

Substitute $-\mathbf{F}_{\text{B on A}}\Delta t$ into the impulse-momentum equation for ball B. Then add the momenta of the two balls. Use the addition property of equality.

$$\begin{aligned} \mathbf{F}_{\text{B on A}}\Delta t &= \mathbf{p}_{\text{A,f}} - \mathbf{p}_{\text{A,i}} \\ \underline{-\mathbf{F}_{\text{B on A}}\Delta t} &\underline{= \mathbf{p}_{\text{B,f}} - \mathbf{p}_{\text{B,i}}} \\ 0 &= (\mathbf{p}_{\text{A,f}} - \mathbf{p}_{\text{A,i}}) + (\mathbf{p}_{\text{B,f}} - \mathbf{p}_{\text{B,i}}) \end{aligned}$$

You can rearrange this result, with the final values on one side and initial values on the other.

$$\mathbf{p}_{\text{A,f}} + \mathbf{p}_{\text{B,f}} = \mathbf{p}_{\text{A,i}} + \mathbf{p}_{\text{B,i}}$$

$$\begin{matrix}\text{total final}\\ \text{momentum}\end{matrix} = \begin{matrix}\text{total initial}\\ \text{momentum}\end{matrix}$$

$$\mathbf{p}_f = \mathbf{p}_i$$

No matter what internal interactions take place in a closed isolated system, its linear momentum never changes. This is called the **law of conservation of linear momentum**.

> *When no net external forces act on a closed system, the total linear momentum of the system remains constant.*

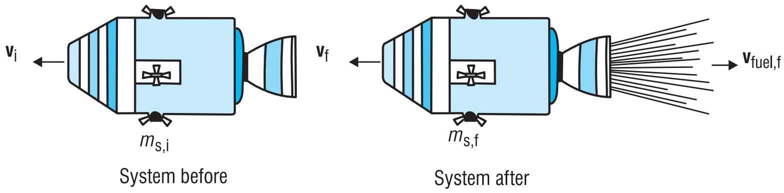
When you use conservation of linear momentum to solve a problem, the first step is to define the closed isolated system. Define the system boundary so: (1) no mass enters or leaves the system (no mass crosses the boundary), and (2) no *external* force acts on the system (all forces are *internal* to the system).

Example 7.4 Conservation of Momentum for a Spaceship

A 670-kg spaceship is traveling at 1200 m/s. The ship's rocket motor burns 120 kg of fuel. After the burn, the fuel exhaust is traveling at 2800 m/s in the direction opposite the spaceship. What is the final speed of the spaceship?

Solution: Define the system as the spaceship (including unburned fuel) and the fuel exhaust. This is a closed system since no mass enters or leaves. No external force is acting on the system, so it is isolated. Therefore, the linear momentum of the system is conserved.

Let m and $\mathbf{v}$ represent the mass and velocity of the spaceship, and m_{fuel} and $\mathbf{v}_{\text{fuel}}$ represent the mass and velocity of the fuel exhaust.



$$\mathbf{p}_f = \mathbf{p}_i$$

$$m_f \mathbf{v}_f + m_{\text{fuel,f}} \mathbf{v}_{\text{fuel,f}} = m_i \mathbf{v}_i$$

The velocities act along a line, so we can use speeds. Let the positive direction be in the direction of the spaceship's velocity. Then $v_{\text{fuel,f}}$ is a negative quantity.

$$m_f v_f - m_{\text{fuel,f}} v_{\text{fuel,f}} = m_i v_i$$

Before the fuel burns, the mass of the spaceship m_i includes the mass of fuel. So $m_i = 670$ kg.

Initially, the fuel and spaceship travel together at the same speed. $v_i = 1200$ m/s.

After the fuel burns, the spaceship mass is 120 kg less.

$$m_f = 670 - 120 = 550 \text{ kg}$$

Substitute the known values into the equation.

$$(550 \text{ kg})(v_f) - (120 \text{ kg})(2800 \text{ m/s}) = (670 \text{ kg})(1200 \text{ m/s})$$

$$v_f = \frac{(670)(1200) + (120)(2800)}{550} \text{ m/s} = 2073 \text{ m/s}$$

The final speed of the spaceship is 2073 m/s.

Summary

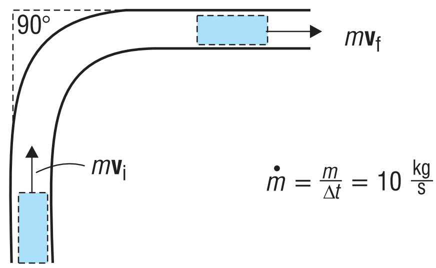
- Linear momentum is the product of an object's mass and its velocity. $\mathbf{p} = m\mathbf{v}$.
- According to Newton's second law of motion, if a net force is exerted on an object, the force equals the object's rate of change of momentum. $\mathbf{F} = \dfrac{\Delta \mathbf{p}}{\Delta t}$.
- When a force is exerted on an object, the impulse is the applied force times the time interval over which it is applied.
- When an impulse is applied to an object, the impulse equals the object's change in momentum. $\mathbf{F}\Delta t = \Delta \mathbf{p}$.
- Newton's third law of motion says that for every action there is an equal and opposite reaction. *Whenever one body exerts a force on another body, the second body exerts an equal and opposite force on the first.*
- The momentum of a closed system is constant if no net external force acts on the system.

Exercises

1. An object that moves in a straight line has linear momentum equal to the product of the object's____________ and ____________.
2. Newton's second law can be written in the form $\mathbf{F} = \Delta \mathbf{p}/\Delta t$. Prove that the SI units for the left side equal those for the right side.
3. The impulse-momentum equation is $\mathbf{F}\Delta t = \Delta \mathbf{p}$. Prove that the English units for the left side equal those for the right side.
4. Find the linear momentum of an 18,000-lb aircraft traveling at 180 mph due west.
5. A car with a mass of 1250 kg moves at a speed of 90 km/h to the north.
   (a) Draw the car's momentum vector. Write the magnitude next to the vector.
   (b) A second car with a mass of 800 kg has the same momentum. What is its velocity?

6. A net force accelerates a 75-kg skier from 1.0 m/s to 6.5 m/s over a time interval of 20 s.
   (a) Sketch the skier's initial and final momentum vectors. What is the skier's change in momentum?
   (b) What is the magnitude of the average net force acting on the skier?
7. An empty truck of mass m_o moves at a speed v_o. Suppose the truck is loaded, so that its mass is doubled, and the speed is reduced by one-half. The truck's new momentum is
   (a) $m_o v_o$.
   (b) $\frac{1}{2}\ m_o v_o$.
   (c) $2\ m_o v_o$.
   (d) $4\ m_o v_o$.
8. Someone kicks a 0.44-kg soccer ball traveling at 8.0 m/s. After the kick, the ball travels at the same speed but in the opposite direction.
   (a) Draw vectors representing the soccer ball's momentum before and after it is kicked.
   (b) Choose a positive direction for momentum. What is the change in momentum of the soccer ball?
   (c) If the ball is in contact with the kicker's foot for 0.75 s, what is the average force exerted on the ball?
9. The law of conservation of linear momentum applies to closed, isolated systems. What two conditions must be true for a system to be closed and isolated?
10. A hockey puck weighs 0.25 lb. A 180-lb goalie catches a puck traveling at 9.5 ft/s. The goalie is initially at rest. At what speed does the goalie slide across the ice after catching the puck?
11. Coal is transported to electrical generating plants in railroad cars. An empty coal car with a mass of 9000 kg coasts at a speed of 2.5 m/s. An 18,000-kg load of coal with no horizontal velocity is dropped into the car. At what speed does the car-coal combination coast?
12. When a 160-lb man steps from a 16-ton commercial fishing boat onto a dock, the only apparent movement is the man's—the boat hardly moves at all. But when the same man steps from a 160-lb fiberglass fishing boat onto a dock, he may fall into the water if the boat isn't tied securely to the dock. Explain why these two situations are different, using the law of conservation of linear momentum.
13. The anvil of a pneumatic nail driver has a mass of 0.30 kg. Air pressure accelerates the anvil from zero to 33.9 m/s in 0.21 s. What average force does the air exert on the anvil?

14. Water flows through a pipe with a 90º elbow, as shown below. The mass flow rate is constant and equals 10 kilograms per second. The speed is also constant and equals 3 meters per second.



Notice that, if you isolate a parcel of water of mass m, the momentum of the parcel before the turn has the same *magnitude* as the momentum after the turn ($mv_i = mv_f$). But the vector momentum changes ($m\mathbf{v}_i \neq m\mathbf{v}_f$) because the direction of the parcel's velocity changes.

(a) Use a parcel of mass $m = 10$ kg. On graph paper, plot the vector momentum $m\mathbf{v}_i$ with a convenient scale.

(b) Plot the vector $m\mathbf{v}_f$ using the same scale. Use the "head-to-tail" method of vector addition to find $m\mathbf{v}_f - m\mathbf{v}_i$. What are the magnitude and direction of the resultant?

(c) The pipe exerts a force on the water to make it turn. What are the magnitude and direction of the force?

15. Two designs for a car bumper are tested for their ability to absorb shock in low-speed collisions. Each bumper is mounted on a 3050-lb car, and the car is rolled into a wall at a speed of 5 mph. Bumper A stops the car in 0.35 s. Bumper B stops the car in 0.78 s.

(a) What is the average force exerted on the car by bumper A?

(b) What is the average force exerted by bumper B?

(c) Below, the magnitude of the force exerted by each bumper is plotted as a function of time. Which graph is bumper A, and which is bumper B?