## 6.3 <br> POWER IN <br> Electrical Systems

## Objectives

- Explain the relationship between power, current, and voltage in electrical systems.
- Explain the relationship between power, current, and resistance in electrical systems.
- Calculate energy usage in kilowatt-hours.
- Solve problems involving power in electrical systems.


## INTERNET connection

To find out more about power in electrical systems, follow the links at www.learningincontext.com.

Electrical systems convert electric work or energy into other forms of work or energy. For example, in a laser electric energy is converted into light. In a radio, electric energy is converted into sound. A sewing machine converts electric energy into motion. An oven converts electric energy into thermal energy. In an electric circuit, a device that converts electric energy into mechanical, thermal, or other electric or magnetic forms is called the load. Power is the rate of energy conversion in the load.


A laser converts electric energy into light.


A sewing machine converts electric energy into motion.

Figure 6.5
Applications of electric power

## Electric Power

A potential difference causes charge to move in the electric circuit of the laser, radio, sewing machine, or oven. A battery, or other source of potential difference for each circuit, creates an electric field. The field is the source of a driving force that does work in pushing charge through the load in the circuit. Then the charge does work as it moves through the load. You learned in Section 2.3 that work done is the product of the charge moved $q$ and the potential difference $\Delta V$ through which the charge moves.

$$
\text { Work }=q \Delta V
$$

Electric power is the rate at which work is done by charge flowing in a circuit.

$$
\text { Pwr }=\frac{\text { work done }}{\text { time interval }}=\frac{q \Delta V}{\Delta t}
$$

Remember, current $I$ is the rate of charge flow in the circuit.

$$
I=\frac{q}{\Delta t}
$$

Therefore, the equation for electric power can be written as the product of the prime mover (potential difference) and a rate (current).

$$
\text { Pwr }=I \Delta V
$$

In electrical systems, power is measured in watts (W), current is measured in amperes (A), and potential difference is measured in volts (V).

$$
\begin{aligned}
1 \mathrm{watt} & =1 \text { ampere } \times 1 \text { volt } \\
1 \mathrm{~W} & =1 \mathrm{~A} \cdot \mathrm{~V}
\end{aligned}
$$

## Example 6.9 Current Through a Laser Power Supply

A circuit in the power supply for a high-power laser produces 1.5 kW when the potential difference across the circuit is 110 V . What is the current flowing in the power supply circuit?

Solution: Solve the power equation for current:

$$
\begin{array}{rlr}
I & =\frac{\mathrm{Pwr}}{\Delta V} & \\
& =\frac{1500 \mathrm{~W}}{110 \mathrm{~V}} & {[1.5 \mathrm{~kW}=1500 \mathrm{~W}]} \\
& =13.6 \mathrm{~A} & {[\mathrm{~W}=\mathrm{A} \cdot \mathrm{~V}]}
\end{array}
$$

The current is 13.6 amps .


Refer to Appendix F for a career link to this concept.

## Power in a Resistance

An electric mixer, a television, a microwave oven, and a hair dryer are loads in a household circuit. As they convert electric energy, they resist charge flow. The resistance $R$ of a load was defined in Section 4.3 as the ratio of the potential difference across the load to the current through the load.

$$
R=\frac{\Delta V}{I} \quad \text { or } \quad \Delta V=I R
$$

Substitute this expression for $\Delta V$ into the power equation.

$$
\begin{aligned}
& \mathrm{Pwr}=I \Delta V=I(I R) \\
& \mathrm{Pwr}=I^{2} R
\end{aligned}
$$

You can also write an equation for power using voltage and resistance. From the definition of resistance, $I=\frac{\Delta V}{R}$. Substitute this expression $I$ into the power equation.

$$
\begin{aligned}
& \mathrm{Pwr}=I \Delta V=\left(\frac{\Delta V}{R}\right) \Delta V \\
& \mathrm{Pwr}=\frac{\Delta V^{2}}{R}
\end{aligned}
$$

All three forms of the power equation are often used in solving problems.

$$
\begin{aligned}
& \mathrm{Pwr}=I \Delta V \\
& \mathrm{Pwr}=I^{2} R \\
& \mathrm{Pwr}=\frac{\Delta V^{2}}{R}
\end{aligned}
$$

Example 6.10 Power Dissipated by a Resistor
A $5.0-\Omega$ resistor is connected across a battery. If 2.2 A of current flow through the resistor,
(a) what power is dissipated by the resistor?
(b) how much energy is dissipated in 12 seconds?


Solution:
(a) $\quad \mathrm{Pwr}=I^{2} R$
$=(2.2 \mathrm{~A})^{2}(5.0 \Omega)$
$[\Omega=\mathrm{V} / \mathrm{A}]$
$=24.2 \mathrm{~W}$
$[\mathrm{W}=\mathrm{A} \cdot \mathrm{V}]$

The resistor dissipates 24.2 watts of power.
(b) Remember, energy dissipation means the resistor converts electric energy into thermal energy. The rate of energy dissipation is power. In a time interval $\Delta t$ of 12 seconds, the energy $E$ dissipated is:

$$
\begin{aligned}
E & =\operatorname{Pwr} \times \Delta t \\
& =(24.2 \mathrm{~J} / \mathrm{s})(12 \mathrm{~s}) \quad[\mathrm{W}=\mathrm{J} / \mathrm{s}] \\
& =290.4 \mathrm{~J}
\end{aligned}
$$

In 12 seconds, the resistor dissipates 290.4 joules.

## Kilowatt-Hours

Electric utility companies are often called "power companies." But they really sell energy, not power. When you use an electrical device you pay for the energy used by the device. This energy equals the rate of energy consumption (the power of the device, in watts) times the number of seconds the device is used.

Remember the relationship between the units of power and energy, $\mathrm{W}=\mathrm{J} / \mathrm{s}$. So a joule is also a watt-second. The electric company could charge you for the number of joules, or watt-seconds, of energy used. But a watt-second is a small amount of energy. For energy sales, electric companies use a larger unit, called a kilowatt-hour (kWh). A kilowatt-hour is the energy consumed by a $1-\mathrm{kW}(1000-\mathrm{J} / \mathrm{s})$ device operated continuously for 1 hour ( 3600 s ).

$$
1 \mathrm{kWh}=(1000 \mathrm{~J} / \mathrm{s})(3600 \mathrm{~s})=3.6 \times 10^{6} \mathrm{~J} \quad \text { or } 3.6 \mathrm{MJ}
$$

Figure 6.6 shows a typical meter that measures household electric energy use in kilowatt-hours. This meter has a small electric motor whose speed of rotation depends on the energy being delivered to the house's circuit. As the motor rotates, it turns a system of gears. The gears rotate pointers on dials, or registers.


Figure 6.6
A kilowatt-hour meter
Once per month the electric company reads the meter to determine the number of kilowatt-hours of energy used since the last reading. The registers are read from left to right. Each register represents a decimal-place value. For example, the right-most register is the one's place, the next register is the ten's place, and so on. The electric company subtracts the reading for last month from the reading for this month, and the difference is the energy used for the month.

## Example 6.11 The Cost of Operating a Television

An electric company charges $\$ 0.085$ per kWh of electric energy consumption. What is the cost of operating a television set for one month ( 30 days) if the set is operated an average of 7.5 hours per day? The television set draws 1.5 A when connected to a house's $120-\mathrm{V}$ circuit.

Solution: Find the energy used in kWh. Energy equals the power times the time interval.

$$
\begin{aligned}
E & =\mathrm{Pwr} \times \Delta t \\
\mathrm{Pwr} & =I \Delta V=(1.5 \mathrm{~A})(120 \mathrm{~V})=180 \mathrm{~W} \text { or } 0.18 \mathrm{~kW} \\
\Delta t & =(30 \mathrm{~d})(7.5 \mathrm{~h} / \mathrm{d})=225 \mathrm{~h} \\
E & =(0.18 \mathrm{~kW})(225 \mathrm{~h})=40.5 \mathrm{kWh} \\
\text { Cost } & =(\$ 0.085 / \mathrm{kWh})(40.5 \mathrm{kWh})=\$ 3.44
\end{aligned}
$$

The cost of operating the TV for one month is $\$ 3.44$.

## Summary

- Power is the rate of doing work. In electrical systems, power can be calculated by multiplying current by voltage: $\mathrm{Pwr}=I \Delta V$.
- In terms of a circuit's resistance, power can be calculated using $\mathrm{Pwr}=I^{2} R$ or $\mathrm{Pwr}=\Delta V^{2} / R$.
- In a device (or load) that converts electrical work or energy into another form, the energy converted is the product of the power and the time interval: $E=\mathrm{Pwr} \times \Delta t$.
- A kilowatt-hour $(\mathrm{kWh})$ is an energy unit used by electric companies.


## Exercises

1. Which of the following equations can be used to calculate power in an electric circuit?
(a) $\mathrm{Pwr}=q \Delta V$
(e) $\operatorname{Pwr}=\frac{\Delta V}{I}$
(b) $\mathrm{Pwr}=\frac{(\Delta V)^{2}}{R}$
(f) $\mathrm{Pwr}=W \Delta t$
(c) $\mathrm{Pwr}=I^{2} R$
(g) $\mathrm{Pwr}=I \Delta V$
(d) $\operatorname{Pwr}=\frac{q \Delta V}{\Delta t}$
(h) $\mathrm{Pwr}=I R$
2. A simple circuit consists of a power source, a resistor, and connecting wires.
(a) Draw a schematic diagram of the circuit.
(b) If the resistance is kept constant and the voltage from the power supply is doubled, how does the power dissipated by the resistor change? Explain.
(c) If the voltage from the power supply is kept constant and the resistance is doubled, how does the power dissipated by the resistor change? Explain.
(d) If both the voltage and resistance are doubled, how does the power dissipated change? Explain.
3. One watt is equal to one volt times one ampere, or $1 \mathrm{~W}=1 \mathrm{~V} \cdot \mathrm{~A}$. Which of the following are also equal to 1 W ?
(a) $1 \mathrm{~V}^{2} / \Omega^{2}$
(e) $1.34 \times 10^{-3} \mathrm{hp}$
(b) $1 \mathrm{~A}^{2} \cdot \Omega$
(f) $1 / 1000$ of a kilowatt
(c) $1 \mathrm{~V}^{2} / \Omega$
(g) $1 \mathrm{cal} / \mathrm{s}$
(d) $1 \mathrm{~J} / \mathrm{s}$
(h) $1 \mathrm{~N} \cdot \mathrm{~m}$
4. A $60-\mathrm{W}$ light bulb and a $100-\mathrm{W}$ light bulb work on a $120-\mathrm{V}$ circuit. Which bulb has a higher resistance? Explain your answer.
5. When an electric toaster works on a potential difference of 110 volts, it draws a current of 9.09 amps . How much power does the toaster use?
6. How much current flows through a 75 -watt light bulb when it operates at 110 volts?
7. A $50-\Omega$ resistor is guaranteed to operate according to Ohm's law up to a maximum power level of 100 W . What is the maximum voltage that should be applied across this resistor?
8. The voltage drop across a $15-\Omega$ resistor is 12 V . How much energy is dissipated by the resistor in 5 minutes?
9. Which uses more energy: a 1-kW electric heater operated for 1 hour or ten 100-W light bulbs left on for 1 hour? Explain.
10. The efficiency of a light bulb is approximately $15 \%$. This means that about $15 \%$ of the energy used by a bulb is converted into light energy. How many joules of energy are emitted as light by a 100-W light bulb in one minute? How many joules are converted into thermal energy in one minute?
11. An electric iron draws 3.2 A of current when connected to a $110-\mathrm{V}$ circuit. How much heat does the iron give off in one hour?
12. An electrical engineer designs a lighting system for an office building. She groups lights into circuits, each protected by a circuit breaker. The maximum current allowed by a breaker in a circuit is 20 amps . Each circuit operates at 277 volts. How many 160-watt florescent lights can be grouped in one circuit?
13. A 5 -hp electric motor operates for 20 hours.
(a) How much energy, in kWh , is consumed?
(b) At $\$ 0.085$ per kWh , how much does it cost to run the motor for 20 hours?
14. A fire department's water boost pump provides a pressure difference of 50 psi at a flow rate of $1200 \mathrm{gal} / \mathrm{min}$. The pump is driven by an electric motor that operates on 600 V . The pump and motor combination is $80 \%$ efficient. How much current does the motor draw? ( $7.48 \mathrm{gal}=1 \mathrm{ft}^{3}$ )
15. The heating element of a 55 -gal electric water heater operates on 220 V and draws 25 A of current. How long will it take to increase the water's temperature from $20^{\circ} \mathrm{C}$ to $60^{\circ} \mathrm{C}$ ? The specific heat of water is $1 \frac{\mathrm{cal}}{\mathrm{g}^{\circ} \mathrm{C}}$ and the density is $1000 \mathrm{~kg} / \mathrm{m}^{3} .\left(1 \mathrm{cal}=4.18 \mathrm{~J}\right.$ and $\left.1 \mathrm{~m}^{3}=264 \mathrm{gal}\right)$
