6.2 Power in Fluid Systems

Objectives

- Explain the relationship between power and work in a fluid system.
- Explain the relationship between a fluid's power, pressure, and change in volume for a constant-pressure process.
- Explain the relationship between a fluid's power, volume, and change in pressure for a steady-flow process.
- Solve problems involving power in fluid systems.

Power is the rate of doing work. You can calculate power by dividing the amount of work W done by the time interval Δt it takes to do the work.

$$\text{Pwr} = \frac{W}{\Delta t}$$

INTERNET *connection*

To find out more about power in fluid systems, follow the links at **www.learningincontext.com.**

Suppose you slide a book across a table at a constant speed. You exert a constant force F on the book for a distance Δd in the direction of the force. The work you do on the book is $F\Delta d$. Your power can be written as follows.

$$\text{Pwr} = \frac{F\Delta d}{\Delta t} = Fv$$

Thus, for a mechanical system, power can be written as the product of the prime mover (force) and a rate (speed). Power can be written as a similar equation for fluid systems.

Fluid Power

Fluids do work when they move objects. The work done is calculated using the fluid pressure and volume. As in Section 2.2, we will consider the work done by fluids, and fluid power, in constant-pressure and steady-flow processes.

Constant-Pressure Processes. A fluid in a cylinder does work on a moving piston. If the fluid pressure P is constant and the volume changes by an amount ΔV, the work done by the fluid is the product $P\Delta V$. Let Δt represent the time interval over which the work is done. The fluid power is the ratio of work to time.

$$\text{Pwr} = \frac{P\Delta V}{\Delta t}$$

The pressure P and time interval Δt are always positive quantities. But volume change ΔV can be positive or negative. If ΔV is positive, work and power are positive—the fluid does work. If ΔV is negative, work and power are negative—work is done on the fluid.

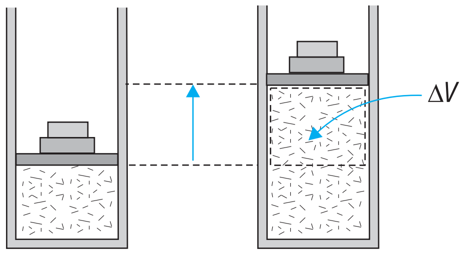


Figure 6.2
An expanding fluid does work on a moving piston. A gas expands at constant pressure if heat is added or if gas is added to the cylinder.

If the fluid in a cylinder is a *gas*, there are two ways to change the volume without changing the pressure:

- Heat transfer—you can heat or cool the gas. The gas expands as the temperature increases and contracts as the temperature decreases.
- Fluid flow—you can add gas to the cylinder or remove gas from the cylinder.

If the fluid in a cylinder is a *liquid*, the change in volume caused by heat transfer is very small unless the liquid changes phase. You can usually neglect the volume change of a liquid caused by heat transfer, unless you are told there is a phase change. Otherwise, there is only one way to change the volume of a liquid without changing the pressure:

- Fluid flow—you can add liquid to the cylinder or remove liquid from the cylinder.

The ratio $\frac{\Delta V}{\Delta t}$ is the rate of change of volume. As in Section 3.2, we write this rate with the symbol $\dot{V}$. If heat is transferred to a gas at constant pressure, $\dot{V}$ is the expansion rate of the gas. If a gas or liquid flows into or out of a region, $\dot{V}$ is the volume flow rate. In SI units, $\dot{V}$ is measured in m^3/s. Substituting this variable into the power equation gives the following.

$$\text{Pwr} = P\dot{V}$$

This form of the power equation is similar to the equation for mechanical power—the product of the prime mover (pressure) and a rate (rate of change of volume).

Example 6.5 Power Required to Fill a Balloon

A tank of compressed air is used to fill balloons. A balloon that is initially flat has a final volume of 1.25 ft^3. The atmospheric pressure is 14.2 psi. What is the power, in horsepower, from the compressed air that is required to fill the balloon in 1.5 s? (1 hp = 550 ft·lb/s)

Solution: The air inflating the balloon does work by pushing against atmospheric pressure. This pressure is constant, so use $\text{Pwr} = P\dot{V}$. Convert the pressure to lb/ft^2.

$$\text{Pwr} = P\dot{V} = P\frac{\Delta V}{\Delta t}$$

$$\text{Pwr} = \left(14.2\frac{\text{lb}}{\text{in}^2}\right)\left(144\frac{\text{in}^2}{\text{ft}^2}\right)\left(\frac{1.25\,\text{ft}^3}{1.5\,\text{s}}\right) = 1704\ \text{ft}\cdot\text{lb/s}$$

Convert to horsepower:

$$\text{Pwr} = (1704\ \text{ft}\cdot\text{lb/s})\left(\frac{1\ \text{hp}}{550\ \text{ft}\cdot\text{lb/s}}\right) = 3.1\ \text{hp}$$

The output of the air is 3.1 horsepower.

Steady-Flow Processes. Fluids flowing through pumps and turbines are examples of steady-flow processes. In a steady-flow process, the mass flow rate is constant. The mass flow rate entering a pump or turbine equals the mass flow rate exiting the pump or turbine. You learned in Section 3.2 that volume flow rate $\dot{V}$ is related to density ρ and mass flow rate $\dot{m}$ as follows:

$$\dot{V} = \frac{\dot{m}}{\rho}$$

If ρ and $\dot{m}$ are both constant, the volume flow rate is constant.

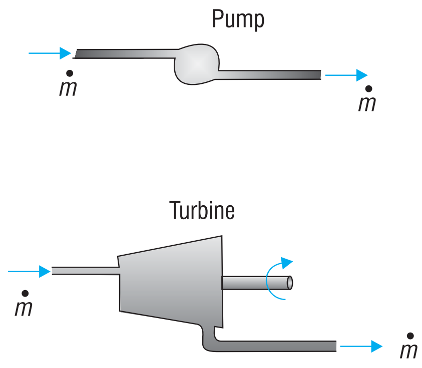


Figure 6.3
A pump does work on a fluid. A fluid does work on a turbine. In each device, the mass flow rate is constant.

Suppose a volume of fluid V moves through a device in a steady-flow process and the fluid pressure changes by an amount ΔP. From Section 2.2, the work done by the fluid is $-V\Delta P$. Let Δt represent the time interval over which the work is done. The fluid power is the ratio of work to time.

$$\text{Pwr} = -\frac{V\Delta P}{\Delta t}$$

The volume V and time interval Δt are always positive quantities. But pressure change ΔP can be positive or negative. If ΔP is positive, work and power are negative—work is done on the fluid (as in a pump). If ΔP is negative, work and power are positive—the fluid does work (as in a turbine).

The ratio $\frac{V}{\Delta t}$ is the volume of fluid moving through a system per unit time. This is the volume flow rate $\dot{V}$. Substituting this variable into the power equation gives the following.

$$\text{Pwr} = -\dot{V}\Delta P$$

Thus, for problems involving steady flow and constant fluid density, the power equation is similar to the equation for mechanical power. But this time it is the product of the change in the prime mover (pressure) and a rate (volume flow rate).

Example 6.6 Power Output of a Pump

A pump transfers jet fuel from a storage tank into an airliner's wing tank through an 8.5-cm-diameter pipe. The fuel travels at an average speed of 3.8 m/s through the pipe. The pump increases the fuel pressure from 150 kPa to 250 kPa. What is the power output of the pump?

Solution: From Section 3.2, the volume flow rate is the product of the pipe cross-sectional area and the fuel speed through the pipe.

$$\dot{V} = Av = \pi r^2 v = \pi\left(\frac{0.085}{2}\,\text{m}\right)^2\left(3.8\,\frac{\text{m}}{\text{s}}\right)$$

$$\dot{V} = 0.0216\ \text{m}^3/\text{s}$$

The power output of the pump equals the power input to the jet fuel.

$$\text{Pwr} = -\dot{V}\Delta P \qquad [\text{Pa} = \text{N/m}^2]$$

$$= -\left(0.0216\,\frac{\text{m}^3}{\text{s}}\right)(250\times10^3 - 150\times10^3)\frac{\text{N}}{\text{m}^2}$$

$$= -2160\ \text{W} \quad \text{or} \quad -2.16\ \text{kW} \qquad [\text{N}\cdot\text{m} = \text{J and W} = \text{J/s}]$$

The negative signs means that work is done on the fuel. The pump power output is 2.16 kilowatts.

Power in Hydraulic and Pneumatic Systems

Hydraulic and pneumatic systems convert power in moving liquids or gases into mechanical power. Figure 6.4 shows a partial hydraulic system. A pump increases the pressure of a working fluid. The fluid moves a piston and load to the right. (A second control valve and additional tubing, not shown in Figure 6.4, reverse the direction of fluid flow. This moves the piston to the left.) In a pneumatic system, air is used as the working fluid. An air compressor replaces the pump, and a pressurized tank replaces the reservoir. The air is usually not recycled.

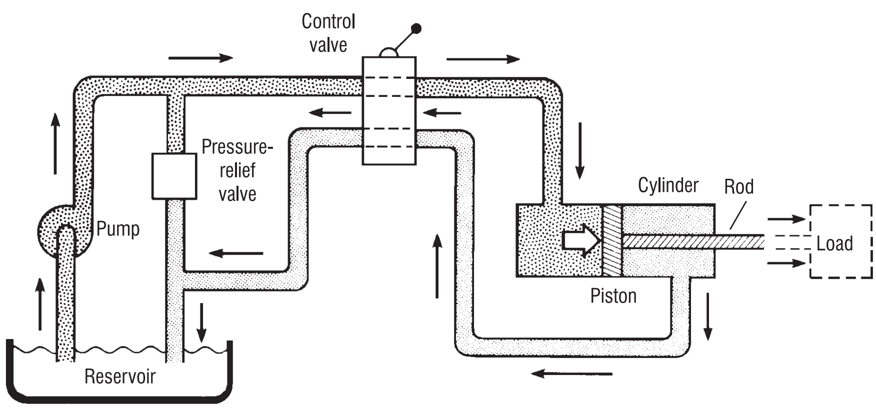
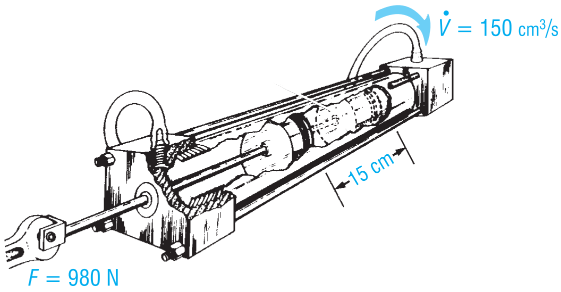


Figure 6.4
A partial hydraulic power system

Example 6.7 Pressure in a Hydraulic System

In a robotic hydraulic power system, a piston moves a 980-N load a distance of 15 cm in 1.2 s. The system's pump delivers high-pressure fluid at a rate of 150 cm^3/s. What is the output mechanical power of the piston? What is the pressure of the fluid?



Solution: The output mechanical power is the work done by the piston per unit time.

$$\text{Power} = \frac{W}{\Delta t} = \frac{Fd}{\Delta t} = \frac{(980\text{ N})(0.15\text{ m})}{1.2\text{ s}} = 122.5\text{ W}$$

The output mechanical power is 122.5 watts. This is also the fluid power, assuming there is no friction between the piston and cylinder and the fluid is nonviscous.

The hydraulic fluid moves the piston at constant pressure. To find the pressure, use the power equation in the form $\text{Pwr} = P\dot{V}$. But first convert the units of volume flow rate to m^3/s.

$$\dot{V} = 150\frac{\text{cm}^3}{\text{s}} \bullet \frac{1\text{ m}^3}{10^6\text{cm}^3} = 1.5 \times 10^{-4}\text{ m}^3/\text{s}$$

Solve the power equation for the pressure P.

$$P = \frac{\text{Pwr}}{\dot{V}}$$

$$= \frac{122.5\text{ J/s}}{1.5 \times 10^{-4}\text{ m}^3/\text{s}} \qquad [\text{W} = \text{J/s}]$$

$$= 8.17 \times 10^5 \frac{\text{N}}{\text{m}^2} \qquad [\text{J} = \text{N} \cdot \text{m}]$$

The fluid pressure is 8.17×10^5 Pa, or 817 kPa.

Power from Heat of Combustion

Internal-combustion engines burn gasoline or diesel fuel to power cars and trucks. In gas turbines, hot gases from burning fuel pass through sets of blades to power airplanes, ships, and electrical generators. Rocket motors burn solid or liquid fuel—and accelerate the hot exhaust gases through nozzles—to provide lifting power for the rocket.

Table 6.1 Heat of Combustion ($MJ = 10^6$ J)

Liquids	MJ/kg
Diesel fuel	44
Gasoline	46
Gases	MJ/m^3
Hydrogen	10
Natural gas	33-71
Acetylene	54
Propane	86

The *heat of combustion* of a fuel is the amount of heat released when one kilogram or one cubic meter of the fuel is completely burned. Table 6.1 lists heats of combustion for several fuels. If you know an engine's power output and rate of combustion of fuel, you can use the heat of combustion to calculate the engine's efficiency.

Example 6.8 Efficiency of a Car Engine

A dynamometer measures the power output of a car's engine as 140 hp. The engine consumes 20.8 kg/h of gasoline. What is the efficiency of the engine?

Solution: The power input to the engine comes from burning gasoline.

$$\text{Input power} = \left(20.8\frac{\text{kg}}{\text{h}}\right)\left(46\times10^{6}\ \frac{\text{J}}{\text{kg}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 2.66\times10^{5}\text{ W}$$

Convert the output power to watts:

$$\text{Output power} = (140\text{ hp})\left(\frac{746\text{ W}}{1\text{ hp}}\right)1.04\times10^{5}\text{ W}$$

The efficiency of the engine is the ratio of output to input power.

$$\text{Efficiency} = \frac{\text{output power}}{\text{input power}} = \frac{1.04\times10^{5}\text{ W}}{2.66\times10^{5}\text{ W}} = 0.39\text{ or }39\%$$

The engine's efficiency is 39%.

Summary

- In a fluid system, power is the rate of work done by the fluid.
- When a fluid expands or contracts at constant pressure, the power output of the fluid is $\text{Pwr} = P\Delta V/\Delta t$.
- In a steady-flow process, a volume of fluid moves through a pressure difference. The power output of the fluid is $\text{Pwr} = -V\Delta P/\Delta t$.
- Fluid power can be positive or negative.

Exercises

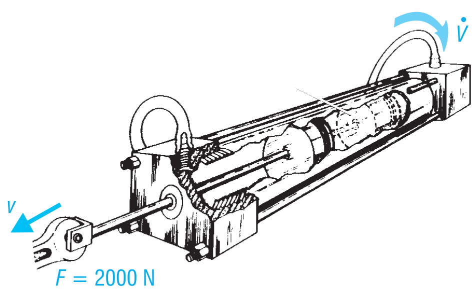
1. What fluid conditions are necessary for you to use the following equation for calculating power?

$$\text{Pwr} = P\dot{V}$$

2. What fluid conditions are necessary for you to use the following equation for calculating power?

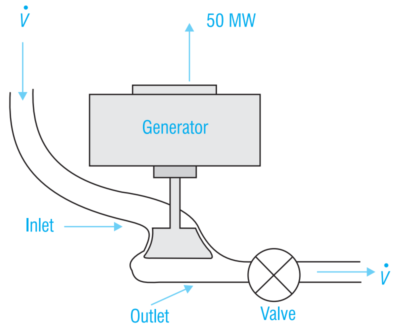
$$\text{Pwr} = -\dot{V}\Delta P$$

3. If fluid power is positive in a process, work is done ____________ (on or by) the fluid. If fluid power is negative, work is done ____________ (on or by) the fluid.
4. You are selecting a pump for a laser cooling system that uses water. The pump must provide a pressure difference of 42 psi and a volume flow rate of 3.5 ft^3/min. You have three pumps from which to choose: ¼ hp, ½ hp, and ¾ hp. Which pump would you select?
5. What flow rate can be expected from a 1-hp pump that produces a pressure difference of 250 kPa?
6. A piston in a hydraulic cylinder has a power output of 7500 watts when it applies a force of 2000 newtons to a load.



   (a) At what speed does the piston move?
   (b) The area of the piston is 40 cm^2. What is the volume flow rate into the cylinder?

7. The inlet to a hydroelectric turbine is located 61 m below the surface of a lake. Water from the lake flows through and turns the turbine. The turbine turns a generator, which produces 50 megawatts of electrical power.



(a) What is the water pressure at the inlet to the turbine? (The density of water is 1000 kg/m^3.)

(b) The water pressure at the outlet of the turbine is 87.2 kPa. If the turbine-generator system is 90% efficient, what is the flow rate of water through the turbine?

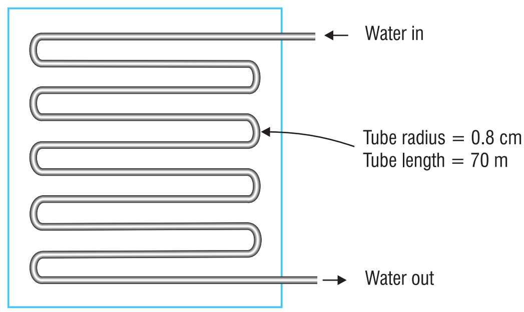
8. In a steam engine, a cylinder fitted with a piston has a volume of 2.0 ft^3 and is filled with steam at 50 psi. Heat is transferred to the steam. In 60 s, the steam expands, pushing the piston at constant pressure until the volume increases to 2.6 ft^3.

(a) What power is developed by the steam during the expansion? Write your answer in units of Btu/s. (1 Btu = 778 ft-lb)

(b) During the expansion, heat is added at a rate of 0.38 Btu/s. At what rate does the internal energy of the steam increase?

9. An in-floor radiant heating system uses warm water to heat a room. The water is circulated by a pump through a tube embedded in the room's sub-floor. The radius and length of the tube are shown in the diagram.

(a) If the volume flow rate of water through the tube is 500 cm^3/s (5×10^{-4} m^3/s), how much does the pressure drop as the water flows through the length of tubing in the room? The viscosity of the water is 6.5×10^{-4} Pa·s. (Remember Poiseuille's law from Section 4.2.)



(b) How much power is needed from the pump to move water through the room?

10. An earthmover's diesel engine consumes 36.6 kg/h of fuel.

(a) What is the power input to the engine?

(b) What is the power output if the power conversion efficiency is 32%?