6.1 POWER IN MECHANICAL SYSTEMS

Objectives

- Explain the relationship between power and work.

- Explain the relationship between power, force, and speed for an object in translational motion.

- Calculate a device's efficiency in terms of the ratio of its output to input power.

- Explain the relationship between power, torque, and angular speed for an object in rotational motion.

- Solve problems involving power in mechanical systems.

Power in Translational Systems

Often the *time* it takes to complete an activity is as important as the work required. For example, when you run up a flight of stairs, you do the same amount of work as when you walk up the stairs. But these two activities are not the same, and you can feel the difference. When you run up the stairs, you are more tired than when you walk. When you run, you do the work *faster*. The rate of doing work or expending energy is **power**. It takes more power to run up the stairs than to walk.
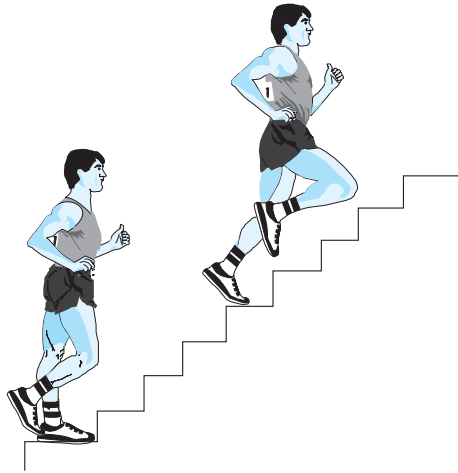
Figure 6.1
When you run up a flight of stairs, you do the same amount of work as when you walk. But you do the work faster. More power is required to run.

If an amount of work W is done in a time interval Δt, the power is the ratio of W to Δt.

$$\text{Power} = \frac{\text{work done}}{\text{time interval}}$$

$$\text{Pwr} = \frac{W}{\Delta t}$$

We use the symbol Pwr as a variable instead of P to avoid confusion with pressure P in the next section.

In the SI system, work and energy are measured in joules (J) and time is measured in seconds (s), therefore, power is measured in joules per second (J/s). One joule per second is called one **watt** (W). 1 watt = 1 W = 1 J/s.

The unit of power in the SI system was named after James Watt, the Scottish engineer who made the steam engine an important power source during the Industrial Revolution. Watt used the English system of measurement, in which work is measured in foot pounds (ft·lb) and time is measured in seconds (s). Therefore, power is measured in ft·lb/s. Steam engines were invented to replace work done by horses and donkeys in pumping water out of coal mines. Watt estimated that a horse could do work at a rate of about 550 ft·lb/s. So he called this rate one horsepower (hp). 1 hp = 550 ft·lb/s.

One horsepower is much larger than one watt. In fact, the watt is a rather small quantity for many applications. The kilowatt (kW) is commonly used. You can use the following factors for converting among units:

$$1 \text{ kW} = 1000 \text{ W}$$

$$1 \text{ hp} = 550 \text{ ft·lb/s}$$

$$1 \text{ hp} = 746 \text{ W}$$

Example 6.1 Power Required to Lift an Elevator

The total mass of a loaded elevator is 1200 kg. An electric motor raises the elevator three floors (15 m) at a constant speed in 12 seconds. What is the power output of the motor?

Solution: Since the elevator travels at a constant speed, no net force is acting on it. The force F_{motor} exerted upward equals the weight, or force of gravity F_g acting downward. The work W done by the motor in raising the elevator a height h is the force times the distance.
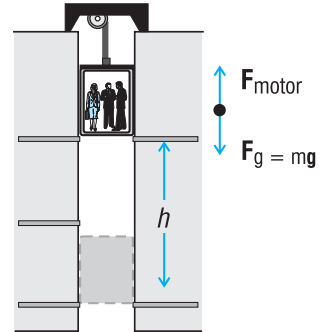
$$W = F_{motor}\, h = mgh$$

Substitute mgh for W in the equation for power:

$$Pwr = \frac{W}{\Delta t} = \frac{mgh}{\Delta t}$$

$$Pwr = \frac{(1200\ \text{kg})(9.80\ \text{m/s}^2)(15\ \text{m})}{12\ \text{s}} \qquad [\text{kg}\cdot\text{m/s}^2 = \text{N}]$$

$$= 14{,}700\ \frac{\text{N}\cdot\text{m}}{\text{s}} = 14.7 \times 10^3\ \frac{\text{J}}{\text{s}} \quad \text{or} \quad 14.7\ \text{kW}$$

The power output of the motor is 14.7 kilowatts.

Suppose a constant force F does work on an object and the force acts in the direction in which the object moves. If the object moves a distance Δd while the force is applied, the work done is the product of force and distance: $W = F\Delta d$. The power delivered to the moving object is

$$Pwr = \frac{W}{\Delta t} = \frac{F\Delta d}{\Delta t}$$

But the speed v of the object is the distance traveled per unit time:

$$v = \frac{\Delta d}{\Delta t}$$

Therefore, when a constant force acts on an object in the direction of motion, the power delivered is the product of the force and speed.

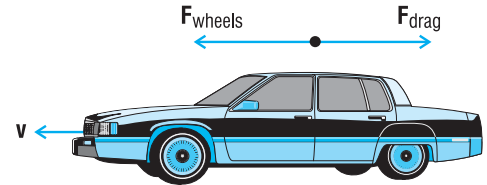$$\text{Power} = (\text{force})(\text{speed})$$

$$Pwr = Fv$$

The following example demonstrates this form of the power equation.

Example 6.2 Drag Force on a Car

A car's drivetrain (engine, transmission, differential, drive shaft, and axle) delivers an average power of 50 hp to the rear wheels of the car when it travels at a constant velocity of 60 mph (88 ft/s). What is the total drag force exerted on the car?

Solution: Since the car travels at a constant velocity, no net force is acting on it. The force F_{wheels} exerted by the wheels in the forward direction equals the force of drag F_{drag} in the opposite direction.

Solve the equation $\text{Pwr} = F_{wheels}v$ for F_{wheels} :

$$F_{wheels} = \frac{\text{Pwr}}{v}$$

$$= \frac{(50 \text{ hp})\left(550\dfrac{\text{ft} \cdot \text{lb/s}}{\text{hp}}\right)}{88 \text{ ft/s}}$$

$$= 313 \text{ lb}$$

The drag force on the car is 313 pounds.

Power and Efficiency

Resistive forces, such as drag and friction, act in the direction opposite a body's velocity. Therefore, they do negative work on the body. As discussed in Section 5.4, this is a *dissipation* of energy. Energy dissipation converts mechanical energy into thermal energy, and lowers the efficiency of engines and simple machines. The amount of energy dissipated per unit time is sometimes called a power "loss."

In Section 2.1, the efficiency of a machine was defined as the ratio of output work to input work.

$$\text{Efficiency} = \frac{\text{output work}}{\text{input work}}$$

You can also calculate efficiency using input and output power. Divide the numerator and denominator by a time interval Δt. Output work per unit time is output power, and input work per unit time is input power.

$$\text{Efficiency} = \frac{\text{output work}/\Delta t}{\text{input work}/\Delta t}$$

$$\text{Efficiency} = \frac{\text{output power}}{\text{input power}}$$

$$\text{Percent efficiency} = \frac{\text{output power}}{\text{input power}} \times 100\%$$

Friction, drag, or electrical resistance converts some of the input power of any real system into *thermal power*. Thermal power is the energy dissipation rate, or the rate of conversion of work or energy into thermal energy. Because of energy dissipation, the output power of a device or system is always less than the input power. The efficiency of a real system is less than 100%.

Example 6.3 Input Power of an Elevator Motor

The elevator motor in Example 6.1 has an efficiency of 89%. What is the electrical input power for the motor?

Solution: From Example 6.1, the output power of the motor is 14.7 kW. Let $\text{Pwr}_{\text{input}}$ represent the input power.

$$\text{Efficiency} = \frac{\text{output power}}{\text{input power}}$$

$$0.89 = \frac{14.7 \text{ kW}}{\text{Pwr}_{\text{input}}}$$

$$\text{Pwr}_{\text{input}} = \frac{14.7 \text{ kW}}{0.89} = 16.5 \text{ kW}$$

The electrical input power for the motor is 16.5 kilowatts.

Power in Rotational Systems

A torque applied to an object in rotation can also do work. In Section 2.1 you learned that work equals the product of the applied torque τ and the angle θ (in radians) through which the object moves.

$$W = \tau\theta$$

If you divide both sides of this equation by the time interval Δt over which the work is done, you get an equation for power.

$$\frac{W}{\Delta t} = \frac{\tau\theta}{\Delta t} = \tau\left(\frac{\theta}{\Delta t}\right)$$

The left side of the equation is the power (work done per unit time). In the right side, the ratio of angle moved through to time is the angular speed of the rotating object, $\frac{\theta}{\Delta t} = \omega$. Therefore, when a constant torque acts on an object in rotation, the power delivered to the object is the product of the torque and angular speed.

$$\text{Power} = (\text{torque})(\text{angular speed})$$

$$\text{Pwr} = \tau\omega$$

The following example demonstrates this form of the power equation.

Example 6.4 Torque Produced by a Spindle Motor

Data are stored magnetically in a computer hard drive on circular platters. A spindle motor rotates the platters so read/write heads (small electromagnets) can read, write, and search for data. If 8.1 W of power are delivered to the spindle motor for reading and writing and the motor rotates at a constant 7200 rpm, what torque is produced by the motor?

Platter

Read/write head

Computer hard drive
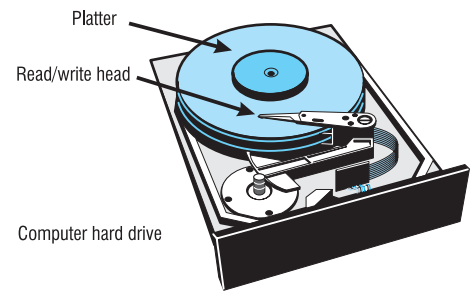
Solution: First convert the angular speed from rpm to radians per second.

$$\omega = \left(7200\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right) = 754\ \text{rad/s}$$

Solve the equation $\text{Pwr} = \tau\omega$ for τ.

Use $\text{Pwr} = 8.1\ \text{W} = 8.1\ \text{J/s}$.

$$\tau = \frac{\text{Pwr}}{\omega}$$

$$= \frac{8.1\ \text{J/s}}{754\ \text{rad/s}} \qquad \text{[Drop the unit } rad.]$$

$$= 0.011\ \text{N}\cdot\text{m} \qquad \text{[J = N}\cdot\text{m]}$$

The spindle motor produces a torque of 0.011 newton·meter.

Summary

- Power is the rate of doing work. Power is measured in watts (W) in the SI system and horsepower (hp) in the English system.

- If a constant force acts on an object in the direction of the object's velocity, the power delivered to the object is the product of the force and the speed. $Pwr = Fv$

- When a device converts input power to output power, the device's efficiency is the ratio of output to input power.

- If a constant torque acts on an object in rotation, the power delivered to the object is the product of torque and angular speed. $Pwr = \tau\omega$

Exercises

1. Which of the following is *not* a unit of power?

    (a) horsepower

    (b) watt

    (c) joule-second

    (d) newton-meter per second

2. How much power is required for you to hold a 1-kg book 1 m above the floor for 100 s?

3. How much power is required for you to run up a flight of stairs to a floor 5.0 m higher than where you start in 6.0 s, if your mass is 50 kg?

4. Which of the following equations for finding power is *incorrect*?

    (a) $Pwr = W/\Delta t$ (c) $Pwr = Fv/\Delta t$

    (b) $Pwr = Fv$ (d) $Pwr = F\Delta d/\Delta t$

5. An electric motor is rated at ¼ hp. What is the power of the motor in ft-lb/s? In watts?

6. An elevator weighs 3500 pounds. It is pulled upward a distance of 30 feet at a constant speed in 10 seconds.

    (a) Find the work done in lifting the elevator.

    (b) Find the speed of the elevator.

    (c) How much power is required to lift the elevator? Write your answer in ft-lb/s and in horsepower.

7. A gas expands in a cylinder, exerting a force of 25 N on a piston. The piston moves at a constant speed of 0.4 m/s.

   (a) What is the power output of the expanding gas?

   (b) How much work does the gas do on the piston in 3 seconds?

8. An electric motor has a shaft torque of 0.55 lb-ft when it turns at a rotational speed of 800 rpm. What is the shaft horsepower of the motor?

9. What is the efficiency of a machine that requires 1200 watts of input power to raise a 900-newton load a distance of 1.5 meters in 11 seconds?

10. To remove a flat tire, you use a hand jack to lift the back of the car 0.5 foot off the ground. The jack lifts 850 pounds.

   (a) What is the power output of the jack if the car is lifted in 10 seconds?

   (b) What is the power output if the car is lifted in 1 minute?

   (c) How much work is done by the jack in part (a)?

   (d) How much work is done in part (b)?

11. A triathlete swims 2.4 miles in 65 minutes with an average power output of 0.22 hp. What is the average drag force exerted on the triathlete during the swim?

12. The bilge pump of a sailboat can lift 7.5 liters of water per minute through a height of 1.5 meters. If the pump is 70% efficient, how much power must be supplied to the pump? (The mass of 1 L of water is 1 kg.)

13. Three equations for power in translational systems are shown below. What are the corresponding analogous equations for power in rotational systems?

| Translational Systems	Rotational Systems
$\text{Pwr} = \dfrac{W}{\Delta t}$	$\text{Pwr} = \,?$
$\text{Pwr} = \dfrac{F\Delta d}{\Delta t}$	$\text{Pwr} = \,?$
$\text{Pwr} = Fv$	$\text{Pwr} = \,?$

14. A dynamometer measures the power output of a car's engine as 185 hp at 4500 rpm. What is the torque output of the engine?

15. A flywheel isolates torque pulsations between a gas compressor and its drive motor. The moment of inertia of the flywheel is 8640 kg·m². The flywheel is accelerated from rest to its operating speed of 441 rpm in 20 seconds.

   (a) What is the change in kinetic energy of the flywheel during its acceleration?

   (b) What is the power input to the flywheel?

16. Prior to production, new car designs are tested for wind drag. The results of a test are shown below. How many horsepower must be delivered to the drive wheels of this car to keep it moving on a level horizontal road at a constant speed of 30 mph? at a constant 60 mph?
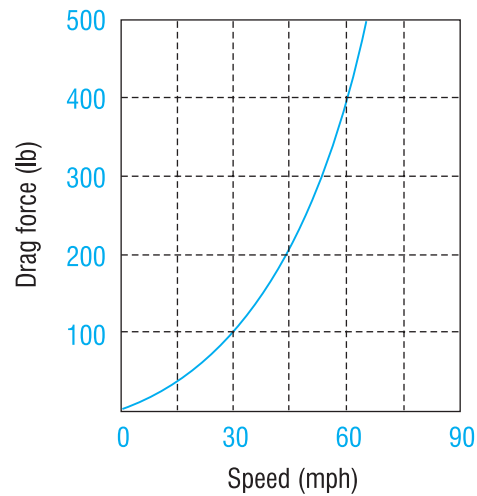
17. The tires of the car in exercise 16 have a radius of 16 inches. How much torque must be applied to the drive wheels to maintain a constant 60 mph? Assume the torque is equally divided between two drive wheels.
   [Hint: You can find the tire's angular speed (in rev/s) by dividing the car's speed (in ft/s) by the distance the car moves per revolution (in ft/rev). This distance is the circumference of the tire.]