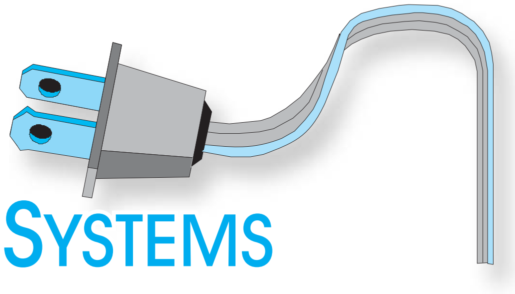
5.3 Energy in Electrical Systems



Objectives

- Describe a capacitor. Explain how a capacitor stores energy.
- Define capacitance.
- Calculate the electrical energy stored in a capacitor.
- Describe an inductor. Explain how an inductor stores energy.
- Define inductance.
- Calculate the electrical energy stored in an inductor.

You learned in Section 2.3 that work must be done to create an electric field. Work is required because there is a coulomb force of attraction between positive and negative charges. A force must be applied to overcome the coulomb force and separate the charges. This force acts over a distance and does work. In an isolated electrical system where energy is conserved, what happens to this work? In mechanical and fluid systems, work can be stored as potential energy, and the same is true in electrical systems. The work is stored as potential energy in the electric field.

INTERNET connection

To find out more about energy in electrical systems, follow the links at **www.learningincontext.com.**

Capacitors

A **capacitor** is an electrical device that stores energy in an electric field. A pair of parallel metal plates, as illustrated in Figure 5.17, is a capacitor.

A power supply, or other source of potential difference, removes electrons from one plate and deposits electrons on the other plate. (We assume the process of moving electrons occurs slowly.) Thus, one plate becomes positively charged and the other becomes negatively charged. An electric field, and potential difference, is established between the plates. Charge separation and the buildup of electric field and voltage continue until the potential difference across the capacitor equals that of the power supply.

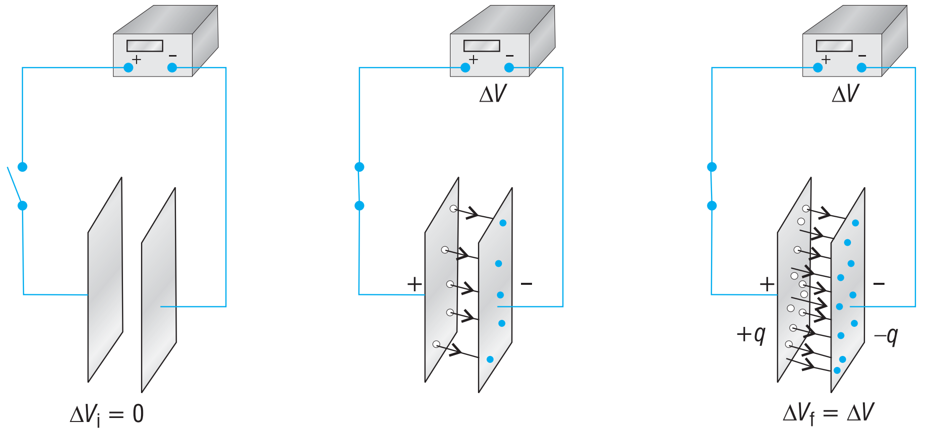


Figure 5.17
As a capacitor is charged, the electric field and potential difference between the plates increase. The work done to establish the electric field equals the potential energy stored in the field.

Before the switch in Figure 5.17 is closed, there is no potential difference between the plates, so $\Delta V_i = 0$. Right after the switch is closed, the work required to move charge is small because there is little existing charge on the plates opposing movement. But, as charge builds up, more work is needed. Let ΔV_f represent the final potential difference across the plates. The average potential difference $\Delta V_{average}$ during the charge transfer is

$$\Delta V_{average} = \frac{\Delta V_f + \Delta V_i}{2} = \frac{\Delta V_f + 0}{2} = \tfrac{1}{2}\Delta V_f$$

If the power supply voltage is ΔV, $\Delta V_f = \Delta V$.

You learned in Section 2.3 that the work W done in an electrical system is the product of the potential difference and charge moved q. But the potential difference is not constant while the capacitor is being charged. So use the average potential difference to calculate work.

$$W = q\Delta V_{average} = q(\tfrac{1}{2}\Delta V_f) = \tfrac{1}{2}q\Delta V$$

Therefore, the work done to separate an amount of charge q and to create a potential difference ΔV across a capacitor is one-half the product of q and ΔV. This work is the amount of potential energy stored in the electric field of the capacitor.

Every capacitor contains two conductors separated by an insulator. The capacitor in Figure 5.17 uses air as the insulator, but most capacitors use thin layers of plastic. The conductors are made of a metal foil. Because the materials are very thin and flexible, capacitors with large areas can be made into very compact shapes by rolling or folding the plates. Some examples are shown in Figure 5.18.

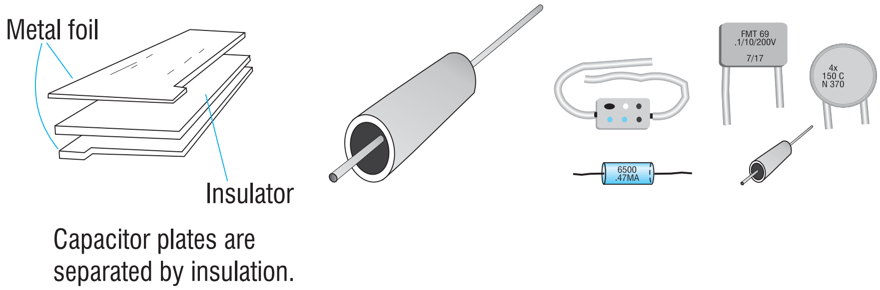


Figure 5.18
Various types of capacitors

Capacitance

Refer to Appendix F for a career link to this concept.

If the charge deposited on a plate of a capacitor is doubled, the capacitor's voltage is doubled. If the charge is halved, the voltage is halved. In other words, the charge q on a capacitor is directly proportional to the potential difference ΔV across the capacitor. The proportionality constant between q and ΔV is called the **capacitance** of the capacitor. We use the symbol C to represent capacitance.

$$\begin{array}{c}\text{Charge on either plate} \\ \text{of a capacitor}\end{array} = (\text{capacitance})\begin{pmatrix}\text{potential difference} \\ \text{between the plates}\end{pmatrix}$$

$$q = C\Delta V$$

The SI and English unit of capacitance is the **farad** (F). Since $C = \dfrac{q}{\Delta V}$, 1 farad = 1 coulomb per volt, or 1 F = 1 C/V.

Remember, one coulomb is a very large amount of charge, so the farad is a very large unit. Most capacitors have capacitances measured in *microfarads* (μF) or *picofarads* (pF).

$$1\ \mu\text{F} = 10^{-6}\ \text{F} \qquad\qquad 1\ \text{pF} = 10^{-12}\ \text{F}$$

Capacitors like those shown in Figure 5.18 have fixed values of capacitance that are labeled on the outside of the capacitors.

Example 5.11 Charge on a Capacitor

A 240-volt motor has a 50-μF capacitor in its starting circuit. When the motor is started, the entire 240 V charges the capacitor. What is the charge on the capacitor after the voltage is applied?

Solution: $q = C\Delta V$

$$= (50 \times 10^{-6}\ \text{F})(240\ \text{V}) = 0.012\ \text{C}$$

Notice that we have used the conversion 1 F = 1 C/V.

When connected to the 240-V potential difference, the capacitor stores 0.012 coulomb of charge.

Potential Energy in Capacitors

The potential energy stored in a capacitor equals the work done in establishing the electric field. At the beginning of the section, we derived the expression $\frac{1}{2}q\Delta V$ for this work.

$$\text{PE} = \tfrac{1}{2}q\Delta V$$

We can use the definition of capacitance to write another form for this equation. Substitute $C\Delta V$ for q:

$$\text{PE} = \tfrac{1}{2}(C\Delta V)\Delta V = \tfrac{1}{2}C\Delta V^2$$

$$\text{Potential energy stored in a capacitor} = \tfrac{1}{2}(\text{capitance})\begin{pmatrix}\text{potential}\\ \text{difference}\end{pmatrix}^2$$

$$\text{PE} = \tfrac{1}{2}C\Delta V^2$$

There is only one set of units for this equation. Capacitance is measured in farads, potential difference is measured in volts, and energy is measured in joules. 1 J = 1 F·V^2.

Example 5.12 Energy of a Capacitor

How much potential energy is stored in the 50-μF capacitor of Example 5.11 after it is charged by the 240-V potential difference?

Solution: $\text{PE} = \frac{1}{2}C\Delta V^2$

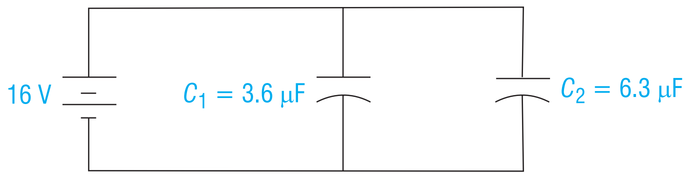
$$= \tfrac{1}{2}(50 \times 10^{-6}\ \text{F})(240\ \text{V})^2 = 1.44\ \text{J}$$

The capacitor stores 1.44 joules of energy.

Notice the symbol used to represent a capacitor in the circuit diagram in Example 5.13 below.

Example 5.13 Capacitors in Parallel

Capacitors of 3.6 μF and 6.3 μF are connected in parallel and charged with a single 16-V source of potential difference. Which capacitor has a greater charge? What is the energy stored in this capacitor?



Solution: Since they are connected in parallel, both capacitors have the same 16-V potential difference. Use the equation $q = C\Delta V$:

$$q_1 = C_1(16\text{ V}) \quad \text{and} \quad q_2 = C_2(16\text{ V})$$

Since $C_2 > C_1$, then $q_2 > q_1$.

The potential energy stored in this capacitor is PE_2:

$$\begin{aligned} PE_2 &= \tfrac{1}{2}C_2\Delta V^2 \\ &= \tfrac{1}{2}(6.3 \times 10^{-6}\text{ F})(16\text{ V})^2 = 8.1 \times 10^{-4}\text{ J} \end{aligned}$$

Magnetic Fields and Induced EMF

An electric field exists around any set of electrical charges. The electric field can be measured using the force exerted on a test charge placed in the field. If the electric charges are moving, they create a **magnetic field** as well. A test charge will experience a force from the magnetic field only if the test charge is moving.

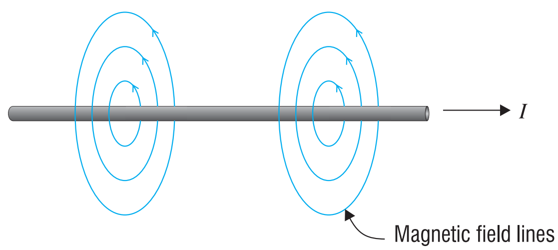


Figure 5.19
Magnetic field lines around a straight wire carrying a current

Figure 5.19 shows the magnetic field around a straight wire that carries a current I. The magnetic field lines are concentric circles with the current at the center. A charge placed in the magnetic field will not feel a force unless the charge moves. And, if the charge moves, it creates its own magnetic field. For example, suppose a second, parallel wire carrying a current is placed in the magnetic field. A force is exerted on the wire from the magnetic field of the first wire. The second wire creates its own magnetic field, which causes a force on the first wire. If the two currents are in the same direction, the wires are attracted. If the currents are in opposite directions, the wires are repelled. (See Figure 5.20.)

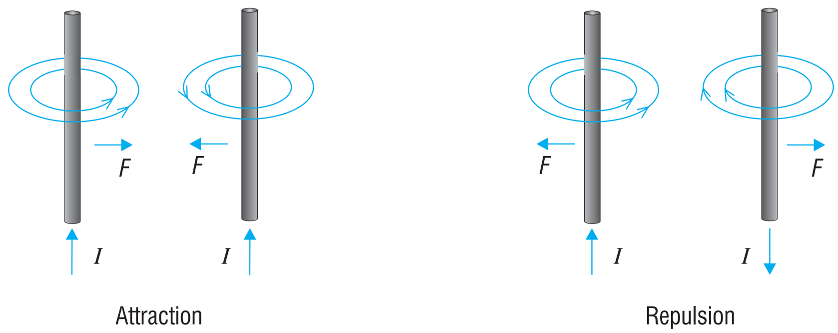


Figure 5.20
Two parallel wires carrying currents create magnetic fields and forces. There is a mutual force of attraction if the currents are in the same direction. The mutual force is repulsion if the currents flow in opposite directions.

If a wire is formed into a loop, the magnetic field has the configuration shown in Figure 5.21(a). A *coil*, or *solenoid*, is a series of loops in the form of a helix. The magnetic field of a coil is shown in Figure 5.21(b). If the loops are close together and the coil is long relative to its diameter, the magnetic field inside the coil is uniform and parallel to its axis except near the ends.

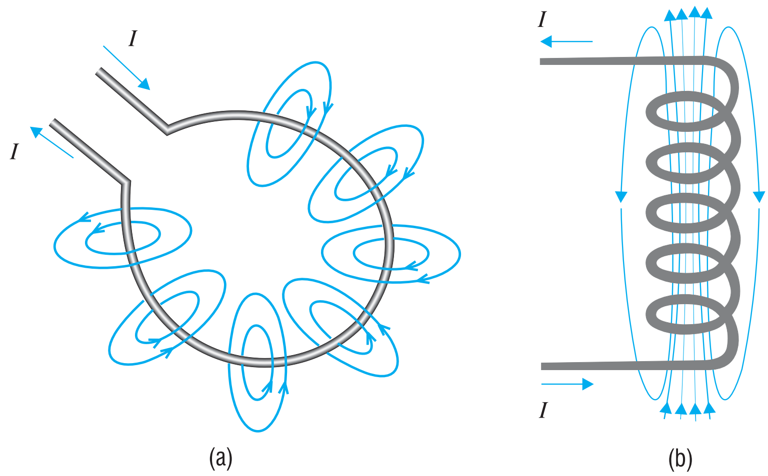


Figure 5.21
Magnetic fields of (a) a wire loop and (b) a coil

A permanent magnet has a magnetic field identical to a coil. This is because the electrons in the atoms of a permanent magnet form current loops and the loops are aligned. The Earth's magnetic field is also caused by current loops in the molten iron core at the center of the Earth. By convention, magnetic field lines point away from the north pole and toward the south pole. Notice the direction of the field lines in Figure 5.22. The Earth's geographic north pole is actually a magnetic south pole.

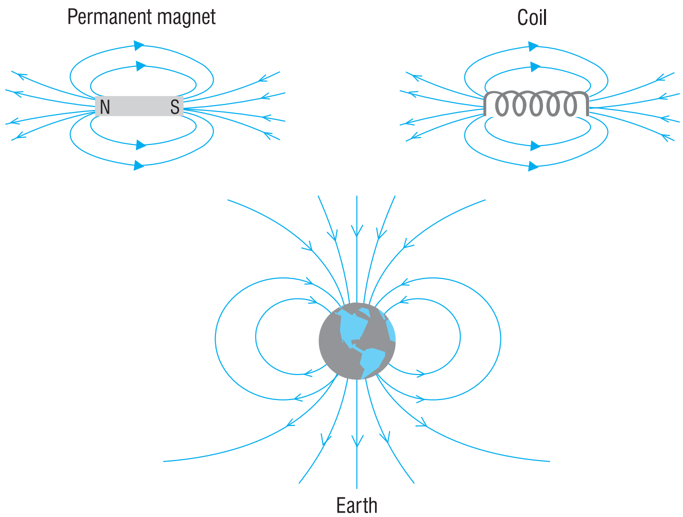


Figure 5.22
The magnetic fields of a permanent magnet, a coil, and the Earth are caused by current loops.

An electric field can cause current to flow and therefore can produce a magnetic field. A magnetic field can also produce an electric field. The process is called **electromagnetic induction**, and it was discovered in 1831 independently by Michael Faraday in England and Joseph Henry in the United States. If a loop of wire moves through a magnetic field (or, equivalently, if a magnetic field moves past a wire), a current is produced in the wire.

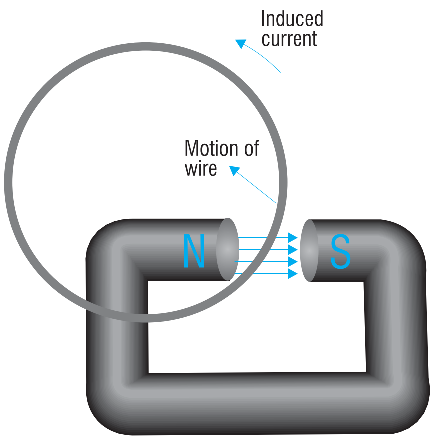


Figure 5.23
Electromagnetic induction

In the 1830s, it was known that a source of electrical energy, such as a battery, is needed to make current flow through a wire. At that time, any source of electrical energy was called an **electromotive force**, or *EMF*. But EMF is not really a force—it is potential difference, or voltage. Even though the term EMF is misleading, it is still used to describe the induced voltage in a wire when a magnetic field moves past the wire or if a wire moves past a magnetic field. An EMF is induced in a wire any time it is in a magnetic field that changes magnitude or direction.

Electromagnetic induction is more important in loops or coils of wire. Figure 5.24 shows a permanent magnet not being moved, being moved toward a coil, and being moved away from a coil. A changing magnetic field inside the coil caused by the movement induces an EMF and current in the coil. You can increase the induced EMF by

- using a more intense magnetic field (larger magnet),
- using a coil with a larger diameter,
- using a coil with more turns of the wire, or
- moving the magnet faster.

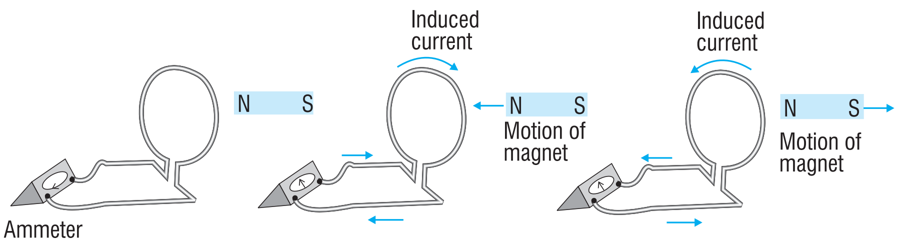


Figure 5.24
A changing magnetic field induces a current and EMF in a coil of wire.

The sign of the EMF and the direction of the current in Figure 5.24 must be consistent with conservation of energy. Energy must be supplied to electrons in the coil to create and increase the induced current. This energy comes from work done in moving the magnet and its magnetic field. Work is required to move the magnet because a force is acting on the magnet, opposing its motion. This force is caused by the magnetic field of the induced current in the coil.

The changing magnetic field caused by the moving magnet induces a current, and the direction of the induced current is such that its own magnetic field opposes the changes responsible for producing it. This statement is called **Lenz's law**, after the Estonian physicist Heinrich Lenz, who described the phenomenon in 1835.

Inductors

An **inductor** is an electrical device that stores energy in a magnetic field. A solenoid or coil of wire, as illustrated in Figure 5.25, is an inductor. Suppose a power supply or battery causes current to flow in the coil. The current generates a magnetic field, shown by magnetic-field lines. As the current increases, the magnetic field increases. The increasing magnetic field induces an EMF in the coil—the same coil whose current generated the field in the first place. By Lenz's law, this *self-induced EMF* opposes the change in magnetic field and current that created the EMF. The self-induced EMF is sometimes called the "back-EMF."

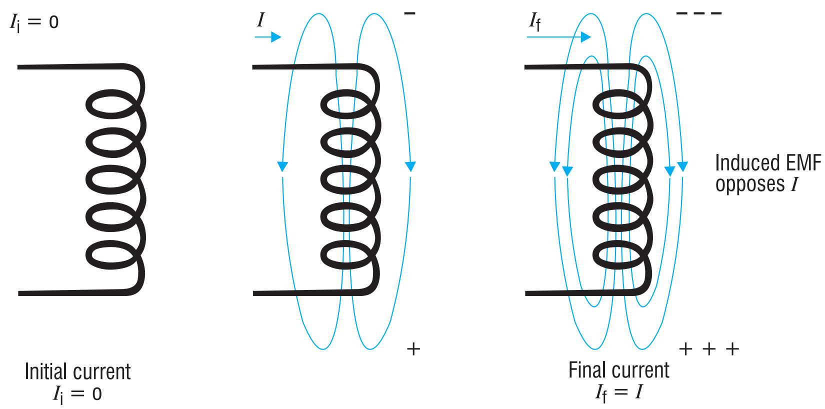


Figure 5.25
As the current in a coil builds up, the increasing magnetic field induces a back-EMF that opposes the buildup. Work must be done to push electrons through the induced EMF.

The back-EMF opposes the current through the coil. Therefore, the power supply or battery must do work to push electrons against the EMF to keep the current flowing. This work is in addition to the work required to push electrons through the wire coil's resistance. The work begins as soon as current begins to flow. Let the current increase steadily from an initial value of 0 A to a final value of I in a time interval Δt. The average current I_{average} during Δt is

$$I_{\text{average}} = \frac{I_f + I_i}{2} = \frac{I + 0}{2} = \tfrac{1}{2}I$$

The total charge q that flows in the coil while the current builds to the final value is

$$q = I_{\text{average}}\,\Delta t = \tfrac{1}{2}I\Delta t$$

The work done by the power supply or battery in moving this charge through the back-EMF is the product of the charge moved and the potential difference, which is the EMF.

$$W = q(\text{EMF}) = \tfrac{1}{2}I\Delta t(\text{EMF})$$

This product is the work done against an inductor's self-induced EMF to create a current I in the inductor. This work is the amount of potential energy stored in the magnetic field of the inductor.

What happens if a current I is flowing in the circuit and the power supply voltage is decreased? The drop in current causes the inductor to develop an EMF to oppose the change. Opposition to a decrease in voltage is an increase—the EMF *adds* voltage to the circuit. The potential energy stored in the inductor's magnetic field is released as work, pushing electrons and slowing the rate of decrease of current.

Inductance

When the current through an inductor changes by an amount ΔI in a time interval Δt, the *rate of change of current* in the inductor is the ratio $\frac{\Delta I}{\Delta t}$. The inductor produces an opposing EMF that is directly proportional to the rate of change of current. If the current changes twice as fast, the EMF is doubled. If the current changes half as fast, the EMF is halved. The proportionality constant between the EMF and $\frac{\Delta I}{\Delta t}$ is called the **inductance** of the inductor. We use the symbol L to represent inductance.

$$\text{EMF} = -(\text{inductance})\left(\begin{array}{c}\text{rate of change}\\ \text{of current}\end{array}\right)$$

$$\text{EMF} = -L\frac{\Delta I}{\Delta t}$$

The minus sign in the equation means that the EMF produced by an inductor is opposite in sign to ΔI. This is required by Lenz's law.

Notice that a steady-state current I does not change. If $\Delta I = 0$, EMF = 0. There is no back-EMF with a steady-state current.

The SI and English unit of inductance is the **henry** (H). Since $L = \frac{\text{EMF}}{-\Delta I/\Delta t}$, 1 henry = 1 volt per ampere per second, or 1 H = 1 V·s/A.

Coils of wire are inductors. The central region of the coil is called the core. An inductor's inductance depends on its geometry and on the core material. For a given current, the higher the magnetic field in the coil, the higher the inductance. The inductance is higher if the coil has more turns of wire, if the coil diameter is larger, or if the coil is longer. If the core of the coil contains a material other than air, its inductance can be changed. An iron core increases the inductance considerably.

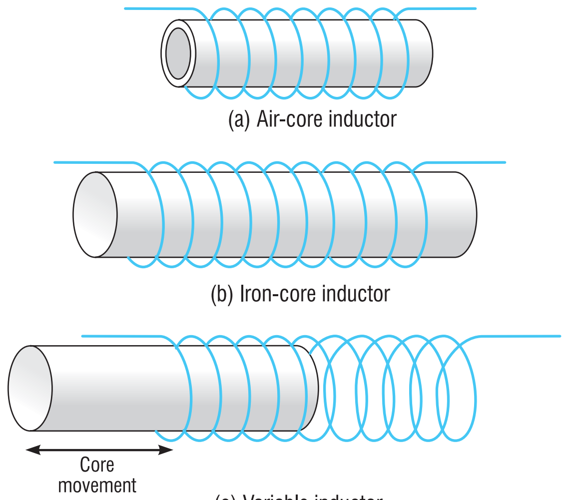


Figure 5.26
Basic inductors

Example 5.14 Current Through an Inductor

A coil with an inductance of 45 mH produces an average opposing EMF of 1.15 V when the current in the coil drops to 0 A in 0.025 s. What is the initial current in the coil?

Solution: Let I_i represent the initial current. Since the current drops to 0, the final current is 0 A. The change in current is $\Delta I = 0 - I_i = -I_i$.

Substitute the given values into the inductance equation.

$$\text{EMF} = -L\frac{\Delta I}{\Delta t}$$

$$\text{EMF} = L\frac{I_i}{\Delta t}$$

$$I_i = \frac{\text{EMF}\ \Delta t}{L} = \frac{(1.15\text{ V})(0.025\text{ s})}{45\times 10^{-3}\text{ H}} = 0.64\text{ A}$$

The initial current through the coil is 0.64 ampere.

Potential Energy in Inductors

The energy stored in an inductor equals the work done during a slow buildup of the current. Earlier in the section, we derived the expression $\frac{1}{2}I(\text{EMF})\Delta t$ for this work.

$$\text{PE} = \tfrac{1}{2}I(\text{EMF})\Delta t$$

We can use the definition of inductance to write another form for this equation. Substitute $L\frac{\Delta I}{\Delta t}$ for EMF. Since $I_i = 0$, $\Delta I = I_f = I$:

$$PE = \tfrac{1}{2} I(L\frac{I}{\Delta t})\Delta t = \tfrac{1}{2} LI^2$$

$$\boxed{\begin{array}{c} \text{Potential energy stored in an inductor} = \tfrac{1}{2}(\text{inductance})(\text{current})^2 \\ PE = \tfrac{1}{2}LI^2 \end{array}}$$

There is only one set of units for this equation. Inductance is measured in henrys, current is measured in amperes, and energy is measured in joules. $1\ J = 1\ H{\cdot}A^2$.

Example 5.15 Energy of an Inductor

A coil has an inductance of 8 H and a resistance of 2 Ω. The coil is connected to a 30-V DC power source. What is the energy stored in the coil when the current is steady?

Solution: First find the steady current I that flows in the coil. From the definition of resistance:

$$I = \frac{\Delta V}{R} = \frac{30\ V}{2\ \Omega} = 15\ A$$

Now use the potential energy equation for an inductor.

$$\begin{aligned} PE &= \tfrac{1}{2}LI^2 \\ &= \tfrac{1}{2}(8\ H)(15\ A)^2 \\ &= 900\ J \end{aligned}$$

The energy stored in the coil is 900 joules.

Controlling Energy in Electrical Systems

A common application of capacitors and inductors is in electronic circuits where voltage changes or current changes need smoothing out. These types of circuits are called "filter circuits." (See Figure 5.27a.)

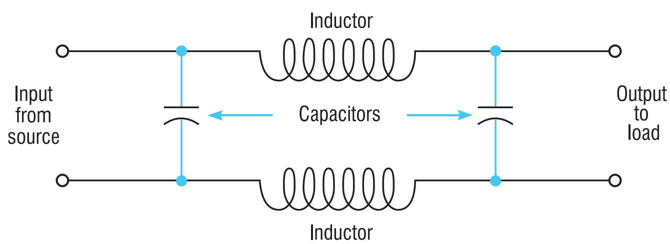
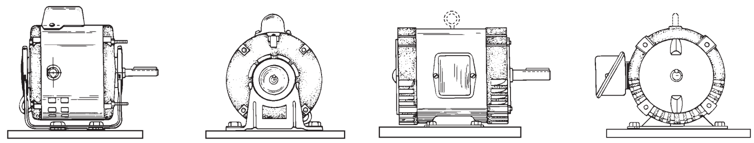


Figure 5.27a
Filter circuit

Frequently, electric motors are equipped with capacitors or inductors to help the motors start running. Motors equipped with capacitor starts (Figure 5-27b) are used to power devices such as air compressors. In this case, the motor must start running, develop enough torque to turn the compressor, and compress air—all at the same time. The energy stored in a capacitor can be used to help the motor do that.

Some electric motors use inductor-starting devices (Figure 5-27c). These types of motors are commonly used to power large machines such as punch presses. Here, the motor must start the rotating mass of the machine moving. This start-up time can be much longer, compared to the “instantaneous” (quick) start-up of an air compressor connected to a motor with a capacitor start.



(b) Capacitor-start motor (c) Induction motor

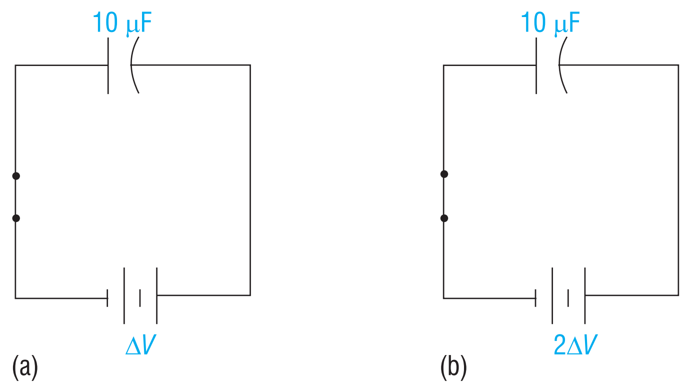
Figure 5.27b,c
Uses of capacitors and inductors

Summary

- A capacitor is an electrical device that stores charge on metal plates separated by an insulator.
- Capacitance is the amount of stored charge per unit voltage across the plates. $C = q/\Delta V$. Capacitance is measured in farads.
- Potential energy is stored in the electric field of a capacitor. $PE = \frac{1}{2}C\Delta V^2$.
- An inductor is an electrical device that creates a magnetic field using a changing current in a loop or coil of wire.
- Inductance is the amount of voltage (or EMF) across the coil per unit rate of change of current through the coil. $L = \dfrac{EMF}{-\Delta I/\Delta t}$. Inductance is measured in henries.
- Potential energy is stored in the magnetic field of an inductor. $PE = \frac{1}{2}LI^2$.

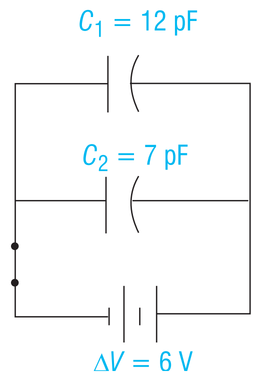
Exercises

1. Where is energy stored in a capacitor? Where is energy stored in an inductor?
2. What SI units are used to measure the following?
   (a) Electric potential energy
   (b) Electric charge
   (c) Electric current
   (d) Electric potential difference
   (e) Electromotive force
   (f) Capacitance
   (g) Inductance
3. A capacitor contains two ______________ separated by ______________.
4. How much charge is deposited on either plate of a 1.5-mF capacitor when the voltage across the capacitor is 10 V?
5. A 10-μF capacitor is connected across a potential difference of ΔV, as in (a) below. A second 10-μF capacitor is connected across a potential difference of $2\Delta V$, as in (b). Which capacitor has a larger amount of charge? Explain the rationale for your answer.



6. What voltage is required to deposit 2.0×10^{-3} C of charge on a 190-μF capacitor?
7. When a capacitor is connected to a 12-V source, it contains 40 μC of charge. What is the capacitance of the capacitor?
8. Energy is stored in a capacitor for the photoflash of a camera. If the capacitance is 12.5 μF and the capacitor is charged with 9.0 V, how much energy is stored?
9. In a circuit, if changes in voltage need to be smoothed out, would you insert a capacitor or an inductor? If changes in current need to be smoothed out, would you insert a capacitor or an inductor? Explain your answers.

10. In the circuit at the right:



(a) What is the potential difference across capacitor C_1? across C_2?

(b) Which capacitor stores less charge?

(c) How much charge is stored on one plate in this capacitor?

(d) How much potential energy is stored in this capacitor?

11. Scientists at Lawrence Livermore National Laboratory produce nuclear fusion with lasers. The lasers use short pulses of energy provided by very large capacitors. One capacitor provides 1.7 MJ of energy when charged with a potential difference of 10 kV. What is the capacitance?

12. Describe how you can use a coil of wire and a bar magnet to generate an electric current.

13. An inductor opposes changes in ________________.

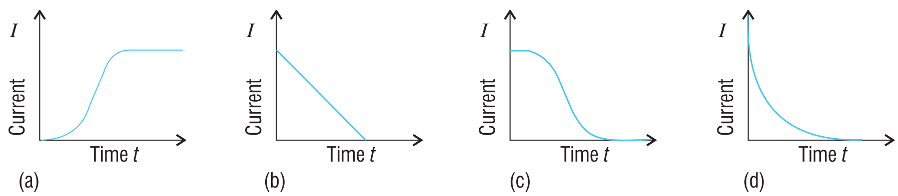
14. What is the voltage across a 15-mH inductor when the current through the inductor changes at a rate of 20 A/s?

15. The current through a coil increases from 1.8 A to 2.4 A in 0.56 s. If the average opposing EMF is 4.0 V, what is the inductance of the coil?

16. How much energy is stored in a 3-H inductor when a steady current of 1.8 A flows through the inductor?

17. A car's starting motor has a solenoid, or coil, that is an inductor. The resistance of the solenoid is 1.2 Ω. It stores 1200 J of energy when the voltage across the solenoid is 10.5 V and the current is steady. What is the inductance?

18. You can make an electromagnet by winding a wire around a nail and connecting the wire ends to a power supply. Which graph below represents a possible relationship between the current through the wire and the time after the power supply is turned on?



19. The resistance of the coil in a large electromagnet is measured with a multimeter. The multimeter uses a D-cell battery source of 1.5 V to measure resistance. The resistance of the coil is 3.8 Ω, and the inductance is 11.7 H. How much energy is stored in the coil as the resistance is measured?