5.1 Energy in Mechanical and Fluid Systems I

Objectives

- Explain the relationship between kinetic energy and motion.
- Calculate the kinetic energy of an object when it is in translational motion and when it is in rotational motion.
- Explain the similarities between the equations for translational and rotational kinetic energies.
- Explain the relationship between the work done on an object or fluid and the change in kinetic energy.
- Use the work-energy theorem to solve problems in mechanical and fluid systems.

INTERNET *connection*

To find out more about energy in mechanical and fluid systems, follow the links at **www.learningincontext.com.**

Refer to Appendix F for a career link to this concept.

Kinetic Energy

An object or a fluid in motion can do work. A moving hammer can do work on a nail. Hot exhaust gas leaving a jet engine can do work on an airplane. Therefore, the moving hammer and exhaust gas have energy. Energy that is due to mass in motion is called **kinetic energy**. Mathematically, kinetic energy KE is the product of one-half the mass m of the object and the square of its speed v.

$$\text{Kinetic energy} = \frac{1}{2}\ (\text{mass})(\text{speed})^2$$

$$KE = \frac{1}{2}mv^2$$

The unit for kinetic energy is the same as the unit for work. In SI, kinetic energy is measured in joules. For example, a 2-kg object moving at 1 m/s has a kinetic energy of $\frac{1}{2}$ (2 kg)(1 m/s)^2 = 1 kg·m^2/s^2 or 1 N·m or 1 J. In English units, energy is measured in foot-pounds.

Kinetic energy increases linearly with mass. Suppose a 1-kg hammer and a 2-kg hammer are moving at the same speed. The 2-kg hammer has twice the kinetic energy of the 1-kg hammer, and can do twice the work.

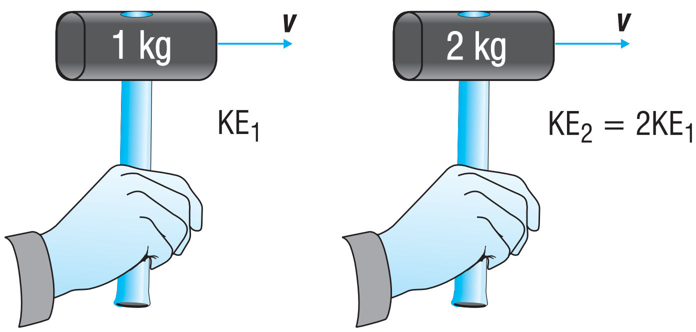


Figure 5.1
A 2-kg hammer has twice the kinetic energy of a 1-kg hammer moving at the same speed.

Kinetic energy increases with the square of the speed. Suppose a 1-kg hammer is moving twice as fast as another 1-kg hammer. The faster hammer has four times the kinetic energy of the slower hammer, and can do four times the work.

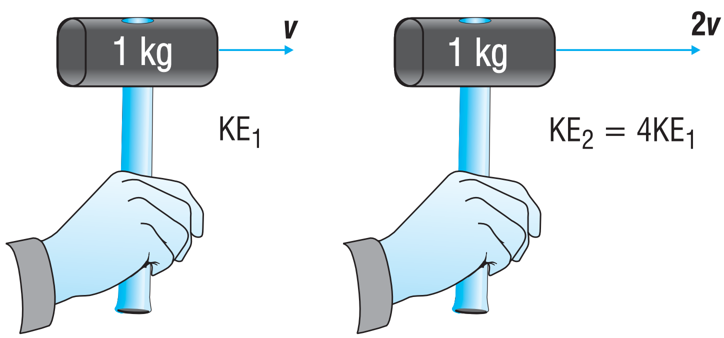


Figure 5.2
A hammer moving twice as fast as an identical hammer has four times the kinetic energy.

Example 5.1 Kinetic Energy of a Volleyball

After a serve, a 0.27-kg volleyball is moving at 22 m/s. What is the kinetic energy of the volleyball?

Solution: $\text{KE} = \frac{1}{2}mv^2$

$$= \tfrac{1}{2}(0.27 \text{ kg})(22 \text{ m/s})^2$$

$$= 65.3 \text{ kg}\cdot\text{m}^2/\text{s}^2 \quad \text{or} \quad 65.3 \text{ J}$$

The kinetic energy of the volleyball is 65.3 joules.

Example 5.2 Kinetic Energy in a Shot

Just before it strikes the ground, an 8.8-lb shot in a shot-put competition is moving at 38 ft/s. What is the kinetic energy of the shot?

Solution: First convert the weight of the shot (pounds) to mass (slugs). You can use the relationship that one slug weighs 32.2 pounds. Or, you can use Newton's second law, with the force of gravity equal to the weight and $a = g$:

$$m = \frac{F}{a} = \frac{8.8 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.273 \text{ lb}\cdot\text{s}^2/\text{ft} \quad \text{or} \quad 0.273 \text{ slug}$$

$$\text{KE} = \tfrac{1}{2}mv^2$$

$$= \tfrac{1}{2}\left(0.273\frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left(38\frac{\text{ft}}{\text{s}}\right)^2$$

$$= 197 \text{ ft}\cdot\text{lb}$$

The kinetic energy of the shot is 197 foot-pounds.

Kinetic Energy in Rotational Systems

A body in circular motion has kinetic energy since it is moving and can do work. For example, a satellite in circular orbit around the Earth has kinetic energy. Let ω represent the satellite's angular speed (in radians per second). Let r represent the distance from the satellite to the center of rotation, which is the center of the Earth.

Figure 5.3
A satellite in circular orbit is a rotational system.

The linear speed v of the satellite is the product of r and ω. If r is measured in meters and ω is measured in rad/s, what are the units of v?

$$v = r\omega$$

The satellite's kinetic energy can be written as follows:

$$\text{KE} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}mr^2\omega^2$$

The product mr^2 is called the **moment of inertia** of the satellite. We use the symbol I to represent moment of inertia. The kinetic energy equation for a body in rotation can be written as follows.

$$\begin{array}{c}\text{Kinetic} \\ \text{energy}\end{array} = \tfrac{1}{2}\left(\begin{array}{c}\text{moment} \\ \text{of inertia}\end{array}\right)(\text{angular speed})^2$$

$$\text{KE} = \tfrac{1}{2}I\omega^2$$

This is the same equation as that for the kinetic energy of a body in translation, but moment of inertia replaces mass and angular speed replaces linear speed.

The moment of inertia of a body in rotational motion is usually not easy to calculate. For the satellite, the equation $I = mr^2$ was based on an important assumption about the mass and its location relative to the axis of rotation. When we said, "let r represent the distance from the satellite to the center of rotation," we did not say what part of the satellite we would measure r from. Instead, we assumed that all the satellite mass was concentrated at a single point, and we measured from that point. This point is called the satellite's *center of mass*. Using the center of mass, you can treat the satellite as a particle—as if it has mass but no size. The motion of the center of mass represents the motion of the entire satellite for rotation around the Earth.

A satellite can also spin, or rotate about an axis through its center of mass. The kinetic energy of this motion is not the same as the kinetic energy of the orbital motion. The equation $KE = \frac{1}{2}I\omega^2$ still holds, but ω is now the angular speed of rotation about the internal axis and I is the moment of inertia about this axis. Calculation of this moment of inertia is more complicated than before, because of the way the satellite's mass is distributed around the axis of rotation.

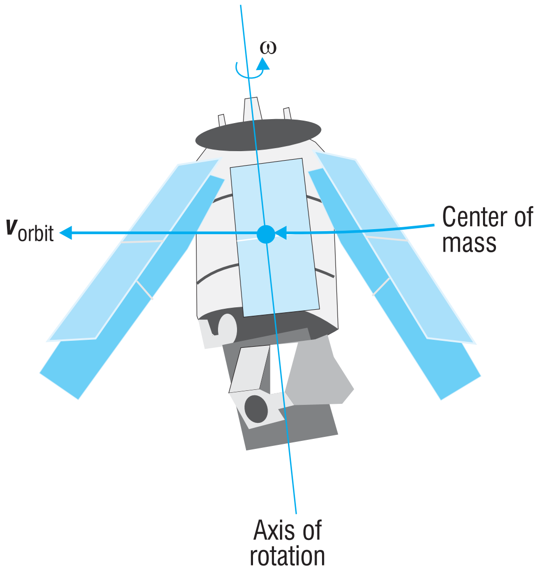


Figure 5.4
A spinning satellite has rotational kinetic energy, not the same as its orbital kinetic energy.

To see how to calculate the moment of inertia of this type of object, look at the compact disc shown in Figure 5.5. Think of all the particles that make up the CD. They have different distances from the axis of rotation, and they may have different masses. Two of these particles are shown in the figure—one on the outside rim of the CD and one on the inside rim. The particles have masses m_1 and m_2, and they are located distances r_1 and r_2 from the axis of rotation.

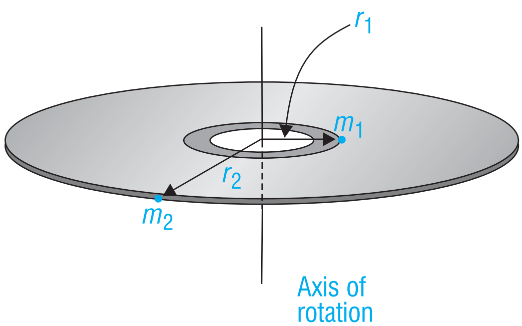


Figure 5.5
The moment of inertia of a CD is the sum of the moments of all the particles that make up the CD.

The moment of inertia of the two particles is the sum $m_1r_1^2 + m_2r_2^2$. But the CD contains many more than two particles, and the moment of inertia of the entire CD is the sum of all the particles' moments:

$$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots$$

The symbol "+ …" means to continue the sum for all particles. It is possible to find this sum using the tools from a branch of mathematics called *calculus*. For an object in the shape of the CD (an annular cylinder), the result is

$$I = \tfrac{1}{2}m(r_1^2 + r_2^2)$$

where m is the total mass of the CD ($m = m_1 + m_2 + m_3 + \ldots$).

Figure 5.6 lists the formulas for moments of inertia for the point particle, annular cylinder, and five other shapes. Each formula contains the mass of the object and a distance (radius or length), but the formulas are different. For each shape, the axis of rotation also is specified. Notice that (d) and (h) are the same shape but the axes are different. The moment of inertia changes when the axis changes.

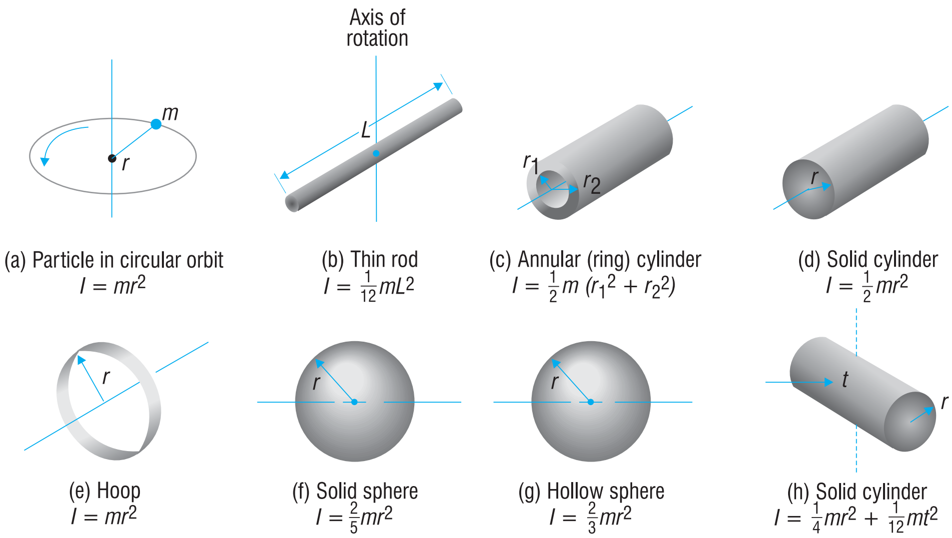


Figure 5.6
Moment of inertia formulas for different-shaped objects. In each case, m is the mass of the object.

Moment of inertia in rotational kinetic energy is analogous to mass in translational kinetic energy, but they are not the same. You can demonstrate the difference with a simple activity. You will need two metersticks, four 100-gram weights, and some masking tape. Tape two weights to one meterstick, at the 40- and 60-cm marks. Tape the other two weights to the other meterstick, at the 10- and 90-cm marks. The total masses of the metersticks are the same, but the moments of inertia are different.

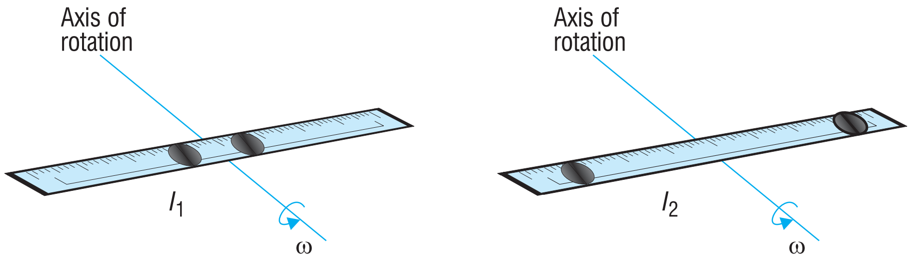


Figure 5.7
The masses are the same, but $I_2 > I_1$. You can tell the difference by rotating each meterstick about an axis through the center of mass.

Hold each meterstick at the 50-cm mark. The center of mass is also at the 50-cm mark. Mass is a property that describes an object's resistance to change in translational motion. Hold each meterstick horizontally, and move it up and down. To they have the same mass?

Moment of inertia is a property that describes an object's resistance to change in rotational motion. Rotate each meterstick about an axis through the center of mass and perpendicular to the length, as shown in Figure 5.7. Which one is harder to rotate? This one has a greater moment of inertia.

Example 5.3 Rotational and Kinetic Energy of a Flywheel

A flywheel is being tested as a possible energy-storage device for automobiles. The flywheel is in the shape of a solid cylinder with a radius of 0.25 m. The mass of the flywheel is 68 kg, and it spins at an angular speed of 6200 rpm. What is the kinetic energy of the flywheel?

Solution: First find the moment of inertia. The formula for a solid cylinder is given in Figure 5.6(d):

$$\begin{aligned} I &= \tfrac{1}{2} m r^2 \\ &= \tfrac{1}{2}(68\ \text{kg})(0.25\ \text{m})^2 \\ &= 2.125\ \text{kg}\cdot\text{m}^2 \end{aligned}$$

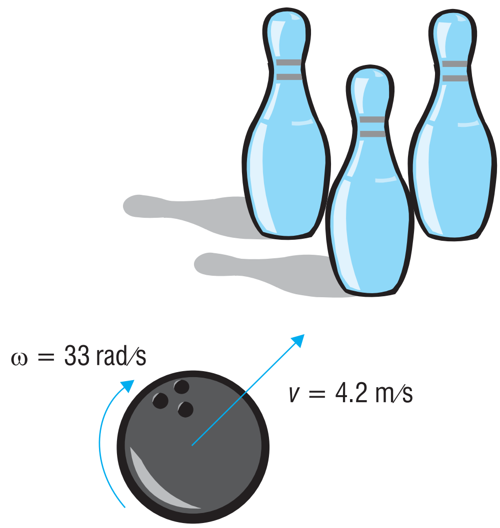
Use this value in the equation for rotational kinetic energy. Convert angular speed to radians per second:

$$\begin{aligned} \text{KE} &= \tfrac{1}{2} I \omega^2 \\ &= \tfrac{1}{2}(2.125\ \text{kg}\cdot\text{m}^2)\left(6200\,\frac{\text{rev}}{\text{min}}\cdot\frac{1\ \text{min}}{60\ \text{s}}\cdot\frac{2\pi\ \text{rad}}{\text{rev}}\right)^2 \\ &= 4.5\times 10^5\ \text{kg}\cdot\text{m}^2/\text{s}^2 \quad \text{or} \quad 4.5\times 10^5\ \text{J} \end{aligned}$$

The kinetic energy of the flywheel is 4.5×10^5 joules.

Example 5.4 Combined Translational and Rotational Kinetic Energies

A 7.2-kg bowling ball has a diameter of 25 cm. When the ball is rolled down an alley, the center of the ball moves at a horizontal speed of 4.2 m/s. The ball rotates at a rate of 33 rad/s. What is the kinetic energy of the bowling ball when it strikes the first pin?



Solution: The bowling ball has translational and rotational motion. The total kinetic energy is the sum of the two kinetic energies:

$$KE = KE_{\text{translational}} + KE_{\text{rotational}} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$

First calculate $KE_{\text{translational}}$:

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}(7.2\text{ kg})(4.2\text{ m/s})^2 = 63.5\text{ kg}\cdot\text{m}^2/\text{s}^2 \quad \text{or} \quad 63.5\text{ J}$$

Now find the moment of inertia of the bowling ball. The ball is a solid sphere. The formula is given in Figure 5.6(f):

$$I = \tfrac{2}{5}mr^2 = \tfrac{2}{5}(7.2\text{ kg})(\tfrac{0.25}{2}\text{ m})^2 = 0.0450\text{ kg}\cdot\text{m}^2$$

Calculate $KE_{\text{rotational}}$:

$$\tfrac{1}{2}I\omega^2 = \tfrac{1}{2}(0.0450\text{ kg}\cdot\text{m}^2)(33\text{ rad/s})^2 = 24.5\text{ kg}\cdot\text{m}^2/\text{s}^2 \quad \text{or} \quad 24.5\text{ J}$$

Find the sum:

$$KE = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = 63.5\text{ J} + 24.5\text{ J} = 88\text{ J}$$

The kinetic energy of the bowling ball is 88 joules.

The Work-Energy Theorem

A hockey puck slides across the ice at an initial speed v_i and with an initial kinetic energy KE_i. A skater strikes the puck and exerts a force in the direction of motion. This force accelerates the puck. The puck's speed increases to a value v_f. Since $v_f > v_i$, the final kinetic energy KE_f is greater than KE_i. Where did the extra kinetic energy come from?

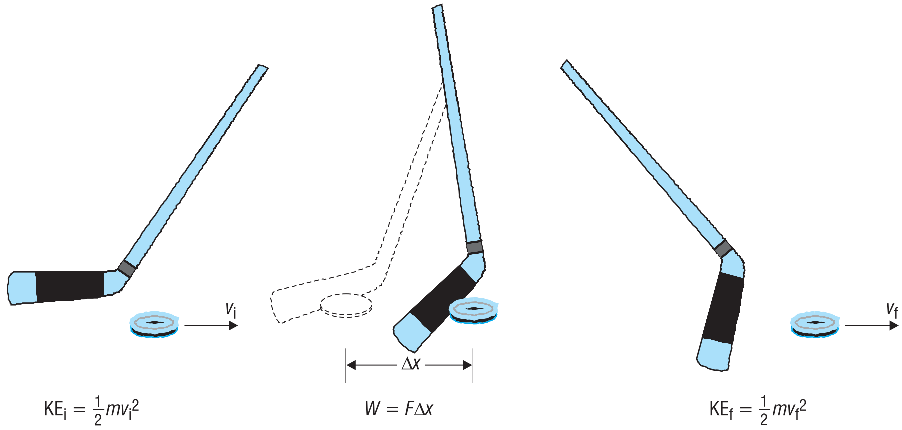


Figure 5.8
Work done on a hockey puck increases its kinetic energy.

The skater applies a force to the puck to accelerate it down the ice. This force is applied over a distance Δx. Therefore, the skater does work on the puck. This work is responsible for the puck's increase in kinetic energy. In general, when work is done on an object, its kinetic energy changes, and the work done equals the change in kinetic energy.

$$W = KE_f - KE_i$$

Or, using the delta symbol:

$$W = \Delta KE$$

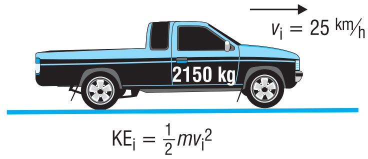
This relationship is called the **work-energy theorem**. The theorem was established in the nineteenth century by the English physicist James Prescott Joule. The SI unit of energy and work is named in his honor.

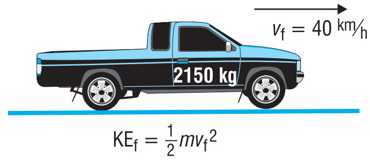
The work-energy theorem establishes an alternative definition of work: a measure of the energy that is transmitted by a force, such as a contact, gravitational, electrical, or magnetic force. In the example above, the work done by the hockey player can be measured by the energy transmitted to the puck (ΔKE).

The work-energy theorem also applies to fluids. Remember, a pressure (or force per unit area) can do work on a fluid ($W = P\Delta V$ or $-V\Delta P$). Work done on a fluid changes the fluid's kinetic energy. You will use the relationship between work and energy for fluids in the next section.

Example 5.5 Work Done on a Pickup

A 2150-kg truck accelerates from 25 km/h to 40 km/h. What work is done by the engine? Neglect friction and drag.





Solution: Convert the initial and final speeds to meters per second:

$$v_i = 25\frac{\text{km}}{\text{h}} \cdot \frac{1000\text{ m}}{\text{km}} \cdot \frac{1\text{ h}}{3600\text{ s}} \qquad v_f = 40\frac{\text{km}}{\text{h}} \cdot \frac{1000\text{ m}}{\text{km}} \cdot \frac{1\text{ h}}{3600\text{ s}}$$

$$v_i = 6.94\text{ m/s} \qquad v_f = 11.1\text{ m/s}$$

Calculate the truck's initial and final kinetic energies:

$$\text{KE}_i = \tfrac{1}{2}mv_i^2 \qquad \text{KE}_f = \tfrac{1}{2}mv_f^2$$

$$= \tfrac{1}{2}(2150\text{ kg})(6.94\text{ m/s})^2 \qquad = \tfrac{1}{2}(2150\text{ kg})(11.1\text{ m/s})^2$$

$$= 5.18 \times 10^4\text{ J} \qquad = 1.32 \times 10^5\text{ J}$$

Now use the work-energy theorem:

$$W = \Delta\text{KE} = \text{KE}_f - \text{KE}_i$$

$$W = 1.32 \times 10^5\text{ J} - 5.18 \times 10^4\text{ J}$$

$$= 8.02 \times 10^4\text{ J}$$

The engines does 8.02×10^4 joules of work.

Summary

- Energy is a property that enables something to do work.
- Kinetic energy is energy that an object has due to its motion.
- An object has kinetic energy if it is in translational motion ($\text{KE} = \frac{1}{2}mv^2$) or if it is in rotational motion ($\text{KE} = \frac{1}{2}I\omega^2$).
- The moment of inertia I is a property that describes an object's resistance to change in rotational motion.
- When work is done on an object, its kinetic energy changes by an amount equal to the work done.

Exercises

1. Is kinetic energy a scalar or a vector quantity? Can the kinetic energy of an object ever be negative? Explain your answers.

2. A bicycle racer and her bike have a combined weight of 128 lb. She pedals the bike 0.35 mile in 1.0 minute at a constant speed. What is her kinetic energy?

3. If you double the speed of an object, by how much do you increase its kinetic energy?

4. Car A is accelerated from 0 to 50 km/h. Car B, which has half the mass of car A, is accelerated from 0 to 100 km/h. Which car's engine does more work?

5. A 2500-kg truck slows from 100 km/h to 72 km/h when the driver applies the brakes. What are the initial and final kinetic energies? How much work is done by the truck's brakes?

6. If the brakes of the truck in Exercise 5 did half as much work, what would be the truck's final speed?

7. If the brakes of the truck in Exercise 5 did twice as much work, what would be the truck's final speed?

8. What quantity in a rotational system is analogous to mass in a translational system? What are the SI units of this quantity?

9. When calculating rotational kinetic energy, what units for angular speed must be used?

10. A 75-gram hailstone has a terminal speed of 180 m/s. How much work can the hailstone do when it hits a shingle on the roof of a house?

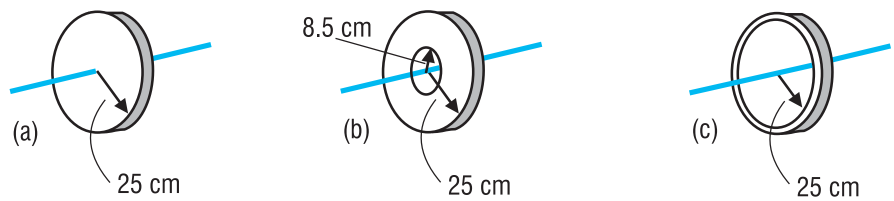
11. If a large airplane has to make an emergency landing, it may have to "dump" (or drain) fuel in the air to reduce its weight. An airliner dumps 20,000 pounds of fuel before landing at a speed of 120 mph. By how much does the airliner reduce its kinetic energy for touchdown by dumping the fuel?

12. A flywheel between an air compressor and its drive motor dampens torque variations. The moment of inertia of the flywheel is 8640 $kg{\cdot}m^2$. What is the kinetic energy of the flywheel when it rotates at 441 rpm?

13. The flywheel in Exercise 12 slows from 441 rpm to 440 rpm in ¼ revolution. Calculate the work done on the flywheel during the ¼ revolution.

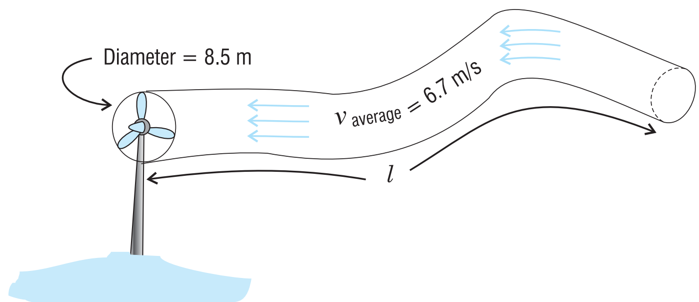
14. An underground water pipeline has a diameter of 0.92 m. The pipeline is 85 km long. If the pipe is completely full of water and the water flows at a speed of 3.1 m/s, what is the kinetic energy of the water? The density of water is 1000 kg/m^3.

15. You are designing the wheels for a solar-powered-car competition. You can use a solid, annular, or hoop design, approximated by the illustrations below. The mass of each wheel is 0.75 kg.



Suppose the car starts from rest and 1 joule of work is transmitted to each wheel, resulting in rotational motion of the wheel. Calculate the angular speed, in revolutions per minute, for each wheel. Which design would you use? Explain your choice.

16. A wind-powered electrical generator has a propeller diameter of 8.5 m. In a 24-hour period, the average speed of air flowing through the generator is 6.7 m/s. You can model this air as that contained in a tube of length l.

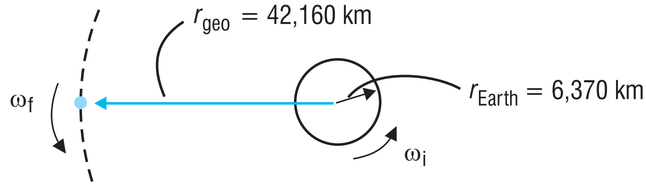


(a) Calculate the length of the tube and the volume of air that flows through the generator in the 24-hour period.

(b) The air has an average density of 1.15 kg/m^3. How much kinetic energy is contained in the wind?

(c) How much energy does the generator extract from the wind during this 24-hour period if it is 30% efficient?

17. A 1000-kg telecommunication satellite is placed in a *geostationary* orbit. In this orbit, the satellite completes exactly one revolution of the Earth in 24 hours. The radius of a geostationary orbit is 42,160 km.



(a) Calculate the moment of inertia of the satellite in the geostationary orbit. Calculate the angular speed of the satellite, in radians per second. What is the satellite's kinetic energy in geostationary orbit?

(b) The satellite is launched from the Guiana Space Center, near the Earth's equator. Before launch, the satellite's angular speed is approximately the same as that calculated in (a). Explain why this is true.

(c) The radius of the Earth is 6370 km. What is the satellite's moment of inertia just before it is launched into space? What is the satellite's kinetic energy just before it is launched into space?

(d) The launch vehicle does work to change the satellite's kinetic energy and to lift the satellite (and its own mass) in the Earth's gravitational field. How much work is done to change the satellite's kinetic energy?