4.4 Resistance in Thermal Systems

Objectives

- Describe a situation in which you would use a material with a high thermal conductivity and a situation in which you would use a material with a high thermal resistance.
- Explain the analogy among fluid resistance, electrical resistance, and thermal resistance.
- Describe the relationship among temperature drop, thermal resistance, and heat flow rate. Use the relationship to solve problems involving heat conduction.
- Calculate thermal resistance for a single layer or multiple layers of different materials.

INTERNET *connection*

To find out more about resistance in thermal systems, follow the links at **www.learningincontext.com.**

Resistance in mechanical systems (friction) opposes the motion of solid objects. In fluid systems, resistance opposes the flow of fluids. Resistance in electrical systems opposes the flow of charge, or current. In thermal systems, resistance opposes the flow of heat.

You learned in Section 3.4 that heat flow rate through a material increases with the thermal conductivity of the material. The bottom of a pan is made of a metal with a high thermal conductivity. This means the heat flow rate will be high when a temperature difference exists between a cooking surface and food inside the pan. You can also say the *thermal resistance* of the bottom of the pan is very low. **Thermal resistance** is a measure of an object's ability to oppose heat transfer.

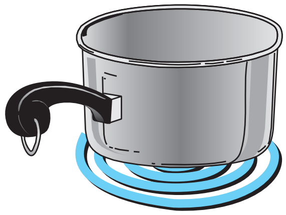


Figure 4.29
A pan has a low thermal resistance or a high thermal conductivity.

Materials with high thermal resistance are used to insulate an object or a region of space. A region is insulated if its surroundings do not affect the region. For example, the walls of a refrigerator have a layer of insulation to increase their thermal resistance. With the insulation, there is a low heat flow rate from the warm air outside to the cold interior of the refrigerator.

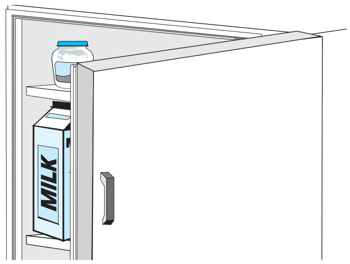


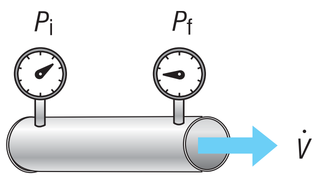
Figure 4.30
The walls and door of a refrigerator have a high thermal resistance.

Thermal Resistance

In Section 4.2, we wrote a ratio for fluid resistance in laminar flow through a tube or pipe. In Section 4.3, we wrote a similar ratio for electrical resistance in charge flow through electrical devices. These ratios compared the prime mover in each system to a flow rate.

For the fluid system, we wrote resistance as the ratio of pressure drop to volume flow rate.

$$R_{\text{fluid}} = \frac{\text{pressure drop}}{\text{volume flow rate}} = \frac{-\Delta P}{\dot{V}}$$



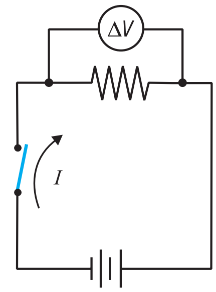
$$\Delta P = P_f - P_i$$

$$\text{Pressure drop} = -\Delta P = P_i - P_f$$

(The negative sign is used to make pressure drop positive. Remember, in the direction of fluid flow, pressure decreases, so ΔP is negative.)

For electrical systems, resistance is the ratio of voltage drop to charge flow rate.

$$R_{\text{electrical}} = \frac{\text{voltage drop}}{\text{charge flow rate}} = \frac{\Delta V}{I}$$

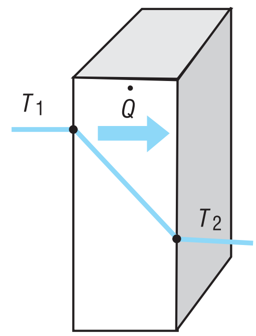


Voltage drop or Potential difference $= \Delta V$

(By convention, a negative sign is not used for voltage drop or potential difference. Remember, ΔV does not mean $V_{\text{final}} - V_{\text{initial}}$.)

In thermal systems, the prime mover is temperature difference and the flow rate is heat flow rate. Therefore, we define thermal resistance as the ratio of temperature drop to heat flow rate.

$$R_{\text{thermal}} = \frac{\text{temperature drop}}{\text{heat flow rate}} = \frac{-\Delta T}{\dot{Q}}$$



$$\Delta T = T_2 - T_1$$

$$\text{Temperature drop} = -\Delta T = T_1 - T_2$$

(The negative sign is used to make temperature drop positive. Remember, heat flows from a high-temperature region to a low-temperature region. ΔT will be negative since $\Delta T = T_{\text{low}} - T_{\text{high}}$.)

Notice the units of thermal resistance

$$R_{\text{thermal}} = \frac{\text{temperature drop}}{\text{heat flow rate}} \begin{matrix} \rightarrow \\ \rightarrow \end{matrix} \frac{°\text{C}}{\text{cal/s}} \text{ or } \frac{°\text{F}}{\text{Btu/s}}$$

When working problems, you can simplify the units in the denominator.

$$\frac{°\text{C}}{\text{cal/s}} = \frac{\text{s}\cdot°\text{C}}{\text{cal}} \qquad \frac{°\text{F}}{\text{Btu/s}} = \frac{\text{s}\cdot°\text{F}}{\text{Btu}}$$

Figure 4.31 shows a slab, or wall, of thickness Δx and composed of a material whose thermal conductivity is k. The area of the slab is A. The left face of the slab has a temperature T_1 and the right face has a temperature T_2, where $T_1 > T_2$. The temperature decreases linearly from T_1 to T_2 through the thickness of the slab.

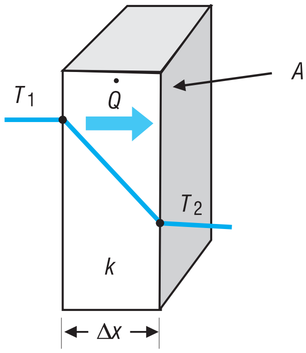


Figure 4.31
Heat flows through a wall from the high-temperature side to the low-temperature side.

You learned in Section 3.4 that heat flows from high temperature to low temperature through the slab, at a rate $\dot{Q}$ given by the heat conduction equation:

$$\dot{Q} = \frac{-kA\Delta T}{\Delta x}$$

Divide both sides of the equation by $-\Delta T$:

$$\frac{\dot{Q}}{-\Delta T} = \frac{kA}{\Delta x}$$

The reciprocal of the left side is R_{thermal}, so find the reciprocal of both sides:

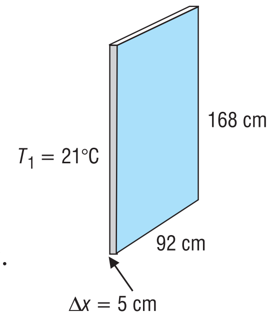
$$R_{\text{thermal}} = \frac{-\Delta T}{\dot{Q}} = \frac{\Delta x}{kA}$$

You can use thermal resistance to solve heat flow problems, just like you use electrical resistance to solve charge flow problems. In electrical circuits, the voltage drop across a resistor is proportional to the current. In heat flow problems, the temperature drop across a slab is proportional to the heat flow rate.

$$-\Delta T = R_{\text{thermal}}\,\dot{Q}$$

Example 4.13 Temperature Inside a Refrigerator

A refrigerator wall measures 92 cm by 168 cm and contains a 5-cm thickness of fiberglass insulation. The heat flow rate through the fiberglass is 4.65 cal/s. If the temperature on the outside of the fiberglass is 21°C, what is the temperature on the inside?



The thermal conductivity of the fiberglass is

$$k = 1.1 \times 10^{-4} \frac{\text{cal} \cdot \text{cm}}{\text{s} \cdot \text{cm}^2 \cdot {}^\circ\text{C}}.$$

Solution: The area of the wall is

$$A = 92 \text{ cm} \times 168 \text{ cm} = 1.55 \times 10^4 \text{ cm}^2.$$

Calculate the thermal resistance:

$$R_{\text{thermal}} = \frac{\Delta x}{k \quad A} = \frac{5 \text{ cm}}{\left(1.1 \times 10^{-4} \dfrac{\text{cal} \cdot \text{cm}}{\text{s} \cdot \text{cm}^2 \cdot {}^\circ\text{C}}\right)\left(1.55 \times 10^4 \text{ cm}^2\right)}$$

$$R_{\text{thermal}} = 2.93 \frac{\text{s} \cdot {}^\circ\text{C}}{\text{cal}} \quad \text{or} \quad 2.93 \frac{{}^\circ\text{C}}{\text{cal/s}}$$

The temperature drop is proportional to the heat flow rate:

$$-\Delta T = R_{\text{thermal}} \dot{Q}$$

$$-(T_2 - T_1) = \left(2.93 \frac{\text{s} \cdot {}^\circ\text{C}}{\text{cal}}\right)\left(4.65 \frac{\text{cal}}{\text{s}}\right) = 13.6^\circ\text{C}$$

$$T_1 - T_2 = 13.6^\circ\text{C}$$

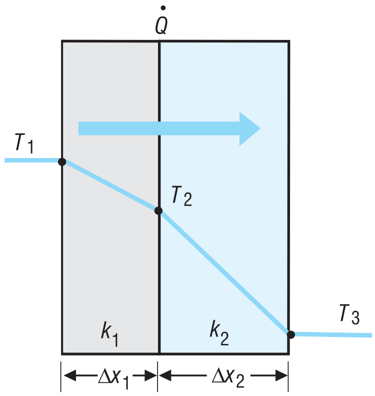
$$21^\circ\text{C} - T_2 = 13.6^\circ\text{C}$$

$$T_2 = (21 - 13.6)^\circ\text{C} = 7.4^\circ\text{C}$$

The temperature inside the refrigerator is 7.4°C.

Thermal Resistance in Series

Walls are often constructed with layers of materials of different thicknesses and different thermal conductivities. For example, Figure 4.32 shows a wall made with two layers. Thermal energy is not created or lost within the wall. Therefore, the heat flow rate through both layers must be the same, and each must equal the heat flow rate through the entire wall. So you can write a single equation for $\dot{Q}$ using the overall temperature drop $-\Delta T_{\text{overall}}$, from T_1 to T_3, and the total thermal resistance R_{total}.



$$R_1 = \frac{\Delta x_1}{k_1 A} \qquad R_2 = \frac{\Delta x_2}{k_2 A}$$

$$\dot{Q} = \frac{-\Delta T_{\text{overall}}}{R_{\text{total}}}$$

$$\text{Overall temperature drop} = -\Delta T_{\text{overall}}$$

$$= -(T_3 - T_1)$$

$$= T_1 - T_3$$

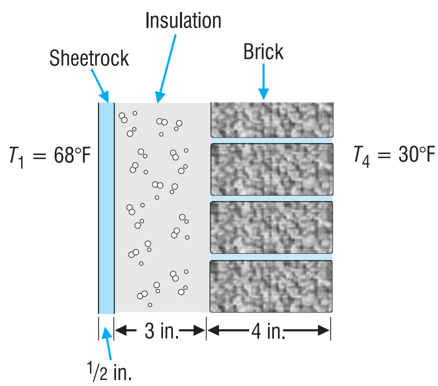
Figure 4.32
The thermal resistance of two materials in series is the sum of the individual resistances.

Heat flows through each resistance, one at a time, just like current flows through each resistance in a series circuit. You can calculate the total thermal resistance in the same way that you calculate total electrical resistance for series circuits—it is the sum of the individual resistances.

$$R_{\text{total}} = R_1 + R_2$$

Example 4.14 Heat Flow Through a Composite Wall

A house wall consists of an inner layer of sheetrock ½ in. thick, a middle layer of insulation 3 in. thick, and an outer layer of brick 4 in. thick. The wall measures 25 ft by 12 ft. The thermal conductivity of each construction material is shown in the table. Calculate the heat flow rate through the wall if the inside temperature is 68°F and the outside temperature is 30°F.



Material	Thermal Conductivity $\frac{\text{Btu} \cdot \text{in}}{\text{h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}}$
Sheetrock	5.2
Insulation	0.32
Brick	4.8

Solution: The area of the wall is

$$A = 25 \text{ ft} \times 12 \text{ ft} = 300 \text{ ft}^2$$

Calculate the thermal resistance of each layer, where $R_{\text{thermal}} = \dfrac{\Delta x}{kA}$:

$$R_1 = \frac{0.5 \text{ in}}{\left(5.2 \dfrac{\text{Btu} \cdot \text{in}}{\text{h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}}\right)(300 \text{ft}^2)} = 0.00032 \frac{\text{h} \cdot {}^\circ\text{F}}{\text{Btu}}$$

$$R_2 = \frac{3 \text{ in}}{\left(0.32 \dfrac{\text{Btu} \cdot \text{in}}{\text{h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}}\right)(300 \text{ ft}^2)} = 0.031 \frac{\text{h} \cdot {}^\circ\text{F}}{\text{Btu}}$$

$$R_3 = \frac{4 \text{ in}}{\left(4.8 \dfrac{\text{Btu} \cdot \text{in}}{\text{h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}}\right)(300 \text{ ft}^2)} = 0.0028 \frac{\text{h} \cdot {}^\circ\text{F}}{\text{Btu}}$$

The total resistance is the sum:

$$R_{\text{total}} = R_1 + R_2 + R_3 = (0.00032 + 0.031 + 0.0028) \frac{\text{h} \cdot {}^\circ\text{F}}{\text{Btu}}$$

$$R_{\text{total}} = 0.034 \frac{\text{h} \cdot {}^\circ\text{F}}{\text{Btu}} \quad \text{or} \quad 0.034 \frac{{}^\circ\text{F}}{\text{Btu/h}}$$

The heat flow rate through the wall is the ratio of overall temperature drop to total resistance:

$$\dot{Q} = -\frac{\Delta T_{\text{overall}}}{R_{\text{total}}} = \frac{-(30 - 68){}^\circ\text{F}}{0.034 \dfrac{{}^\circ\text{F}}{\text{Btu/h}}}$$

$$\dot{Q} = 1118 \text{ Btu/h}$$

Round to two significant figures. The rate of heat flow through the wall is 1100 Btu per hour.

Insulation, or high thermal resistance, is important when you want to keep the temperature of something in a container constant, or to minimize the heat flow rate through a container. In addition to insulating refrigerators and homes, high thermal resistance is used to insulate water heaters, water pipes, and thermos bottles. Water supply and sewer pipes are buried, so they are insulated by soil. A *dewar* is an insulated container for holding extremely cold materials such as liquid nitrogen. Insulation in a dewar is provided by a vacuum. Can you explain why a vacuum is a very good insulator?

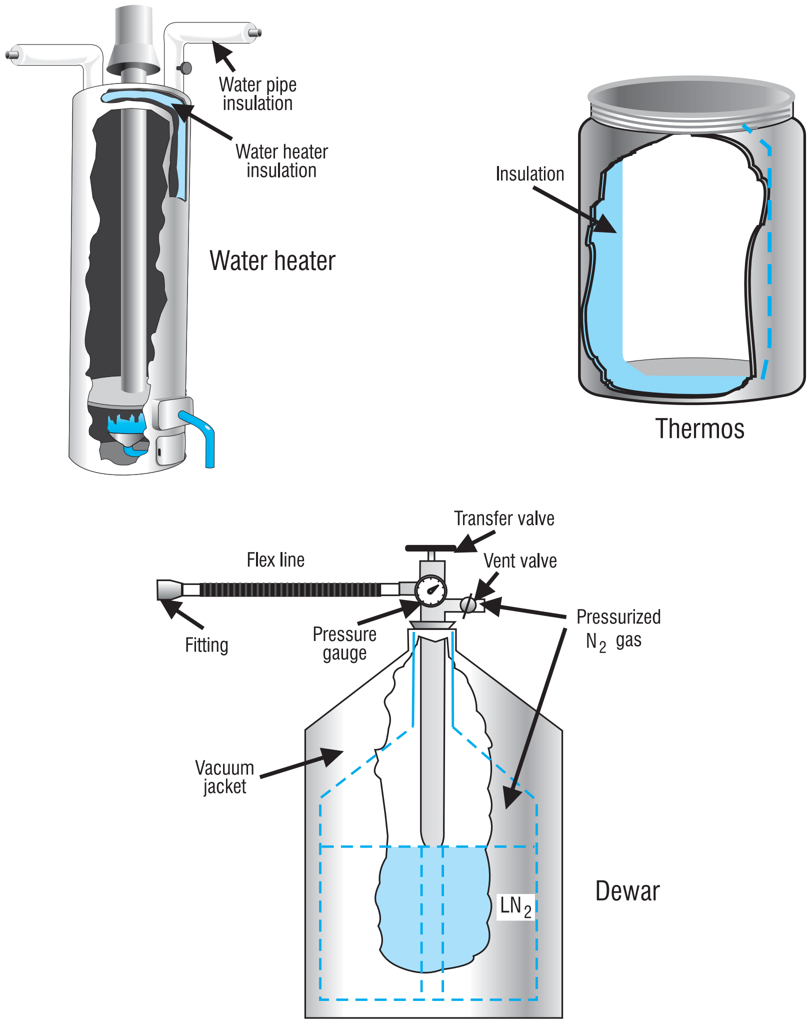


Figure 4.33
A water heater and connecting pipes, thermos, and dewar are insulated to reduce heat flow rates.

Summary

- Materials with high thermal resistance are used for thermal insulation. The heat flow through these materials is low.
- Fluid resistance, electrical resistance, and thermal resistance are defined as the ratio of the prime mover in each system to a flow rate.
- In thermal systems, the temperature drop between two points is proportional to the heat flow rate between the points. The constant of proportionality is the thermal resistance.
- When a wall or slab has multiple layers of different materials, the total thermal resistance for the slab is the sum of the resistances of the layers.

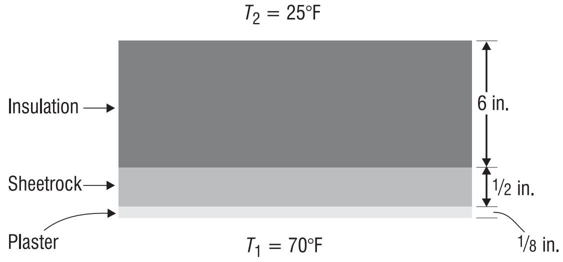
Exercises

1. Thermal resistance is the opposition to the flow of ____________.
2. Thermal resistance is the ratio of ____________ to ____________.
3. In the definition of thermal resistance, explain why the temperature drop $(-\Delta T)$ is a positive quantity.
4. If temperature is measured in °C and heat flow rate is measured in cal/s, what are the units of thermal resistance?
5. A thermos bottle is surrounded by a material with a high thermal resistance. Explain why the following are equivalent reasons for using the thermos bottle to store hot chocolate:

   It will keep the temperature of the hot chocolate high for a long time.

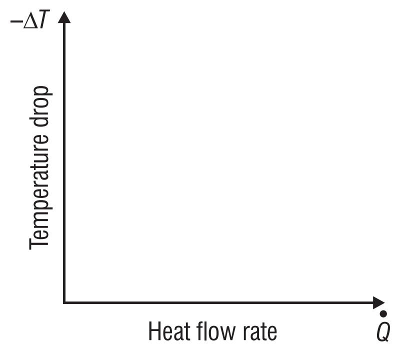
   It will keep the heat flow rate low.
6. The thermal conductivity of a material depends on
   (a) thickness of the material
   (b) type of material
   (c) temperature difference across the material
   (d) surface area of the material
7. The thermal resistance of a building is 0.05 $\frac{\text{h} \cdot {}^\circ\text{C}}{\text{kcal}}$. What is the heat flow rate from the building if the temperature inside is 20°C and the temperature outside is 5°C?
8. An architect designs an office building with large windows. Each window has length L, width W, and thickness Δx. How can the architect reduce the heat flow rate through each window?
   (a) Use a material with a higher thermal conductivity.
   (b) Increase L and/or W.
   (c) Increase Δx.
   (d) All of the above.
9. A window in Exercise 8 measures 10 feet by 12 feet and is 1 inch thick. It is made with a material whose thermal conductivity is 3.2 $\frac{\text{Btu} \cdot \text{in}}{\text{h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}}$.
   (a) What is the thermal resistance of the window?
   (b) What is the heat flow rate through the window if the inside temperature is 70°F and the outside temperature is 0°F?

10. A house ceiling consists of an inner layer of plaster (1/8 in. thick) over a layer of sheetrock (1/2 in. thick) with an outer layer of insulation (6 in. thick). The total surface area of the ceiling is 2500 ft^2. Calculate the heat flow rate through the ceiling if the inside temperature is 70°F and the temperature above the insulation is 25°F. The thermal conductivity of each material is shown in the table.



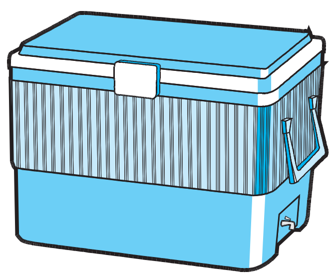
Material	Thermal Conductivity $\frac{\text{Btu} \cdot \text{in}}{\text{h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}}$
Insulation	0.28
Sheetrock	5.2
Plaster	6.0

11. Draw a graph showing the relationship between temperature drop ($-\Delta T$) and heat flow rate ($\dot{Q}$) for the ceiling in Exercise 10. Use the graph to estimate the temperature drop across the ceiling for a heat flow rate of 3000 Btu/h.

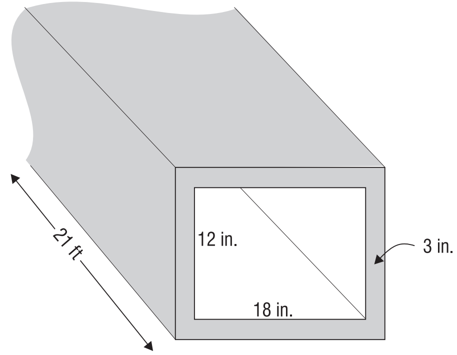


12. A polyurethane foam cooler measures 3 ft × 2 ft × 1.5 ft (inside dimensions). The thermal conductivity of the foam is 0.20 $\frac{\text{Btu} \cdot \text{in}}{\text{h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}}$.

The thickness of the sides, bottom, and top of the cooler is 1.5 inches. The cooler is filled with canned drinks, water, and ice. What is the heat flow rate into the cooler when the outside temperature is 95°F? Assume the temperature is 32°F on all inside surfaces, and 95°F on all outside surfaces.



13. An air-conditioning duct has a rectangular cross section measuring 12 inches by 18 inches. It is wrapped with a fiberglass insulation blanket 3 inches thick. The duct is 21 feet long. Refrigerated air flowing through the duct has an average temperature of 42°F. The duct runs through a ceiling space that has a temperature of 110°F. The thermal conductivity of the insulation is 0.32 $\frac{\text{Btu} \cdot \text{in}}{\text{h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}}$.



(a) As shown in the cutaway diagram above, the duct consists of four walls, with each wall measuring 21 feet in length. Calculate the total area of the inside surface of the duct.

(b) Heat flows through each wall of the duct from the high-temperature region outside to the low-temperature region inside. What is the total heat flow rate from the ceiling space into the duct?

(c) What thickness of insulation would be required to reduce the heat flow rate to 1/3 the value found in (b)?

14. The living quarters for a research group near the Arctic Ocean are elevated, so they are not in contact with the permafrost. This allows cold air to circulate under the buildings to remove heat, which would otherwise melt and damage the permafrost. Each building is in the shape of a box (or rectangular prism), with dimensions 20 ft by 40 ft by 8 ft. The walls, floor, and ceiling of the building are insulated with 12 inches of a material whose thermal conductivity is 0.32 $\frac{\text{Btu} \cdot \text{in}}{\text{h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}}$. You can neglect the insulation value of the other construction materials.

(a) Calculate the total surface area of the walls, the floor, and the ceiling of a building.

(b) The heater in a building maintains a constant indoor temperature of 50°F, while the outside temperature is –40°F (no wind chill). How many Btu per hour must the heater produce?

(c) One of the buildings has a heater with an output of 10,000 Btu/h. How cold can it get outside the building and maintain 50°F inside the building?