4.3 Resistance in Electrical Systems

Objectives

- Explain the differences among conductors, insulators, and semiconductors.
- Define electrical resistance.
- Solve problems using resistance, voltage, and current.
- Describe a material that obeys Ohm's law.
- Calculate the resistance of a wire, given the resistivity of the material and the size of the wire.
- For circuits with resistors in series and in parallel, calculate total resistance, current through each resistor, and voltage drop across each resistor.

INTERNET connection

To find out more about resistance in electrical systems, follow the links at **www.learningincontext.com.**

In the last lesson you learned that, when fluid flows through a pipe, there is resistance to the flow. The amount of fluid resistance depends on the size of the pipe and the material composition of the fluid. Similarly, when charge flows through a wire, there is resistance to the flow. The amount of resistance depends on the size of the wire and its material composition. Electrical resistance of materials and devices varies over a wide range of values—more than any other physical property.

Conductors, Insulators, and Semiconductors

All substances can be ranked by their ability to conduct electric current. For example, charge can flow readily through conductors. Metals, some liquids, and plasmas are good **conductors** because they contain many free electrons. These electrons move easily through the material if an electric field (or potential difference) is applied. In a conductor, electrons flow easily, but there is always some electrical resistance that inhibits flow. However, in some materials resistance disappears at very low temperatures. These materials are called *superconductors*.

Most nonmetallic solids (such as wood, plastic, glass, rubber, and minerals) are **insulators**. In these materials, electrons are tightly bound to atoms or molecules and cannot move easily from place to place. Insulators conduct very small currents even when large potential differences are applied.

Some substances (such as silicon, germanium, gallium, and arsenic) are intermediate in their ability to conduct charge. These are called **semiconductors**. In pure form, semiconductors are better insulators than conductors. But if small amounts of impurities are added (about one atom in ten million) semiconductors become good conductors. Microprocessors and computer chips for data storage are made with semiconductor materials.

Electrical Resistance

A copper wire is a conductor. Free electrons move throughout the wire, but not in straight lines. The electrons continuously bump into copper atoms and other electrons. Each collision causes the electrons to change direction. If a potential difference is applied between the ends of the wire, the electrons are accelerated between collisions away from the negative end of the wire toward the positive end.

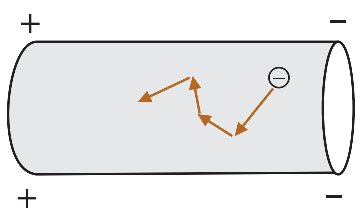


Figure 4.22
Electrical resistance to charge flow is caused by collisions in the wire.

When faster-moving electrons collide with metal atoms, they transfer energy to the atoms and slow down. Energy transferred from the free electrons to the metal atoms increases the wire's internal energy, and its temperature. These collisions between electrons and atoms inhibit the free flow of electrons. This is the source of electrical resistance to current.

Figure 4.23 shows the similarities between an electric circuit and a closed fluid system. In the fluid system, a pump is the source of pressure difference (the prime mover in a fluid system). If you increase the pressure difference produced by the pump, you will increase the fluid flow rate in the system. In the electric circuit, a power supply is the source of potential difference (the prime mover in an electrical system). If you increase the potential difference, you will increase the charge flow rate, or current.

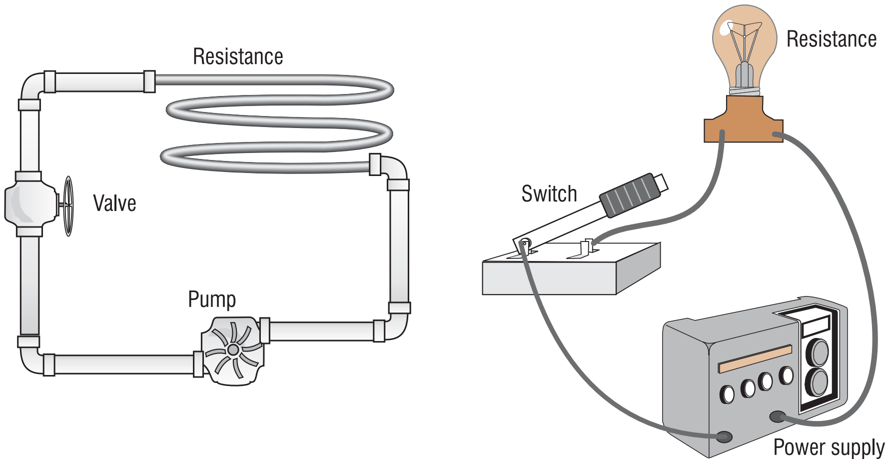


Figure 4.23
Similarities between a closed fluid system and an electric circuit

In the fluid system above, the fluid flows through a long, thin tube. This tube creates drag, or resistance to the flow. The pressure drops as the fluid flows through the resistance. (What makes up for the pressure drop?) In the electrical system, a lamp creates resistance to the current. The voltage drops as charge flows through the resistance. (What makes up for this voltage drop?)

All pipes and tubes resist fluid flow, and all electrical devices resist charge flow. In the last section, we defined a fluid resistance with a ratio involving the prime mover and a flow rate:

$$\text{Fluid resistance} = \frac{\text{pressure drop}}{\text{volume flow rate}}$$

Electrical resistance is defined with a similar ratio:

$$\text{Electrical resistance} = \frac{\text{voltage drop}}{\text{charge flow rate}}$$

Charge flow rate is current. Therefore, **electrical resistance** R is defined as the ratio of the potential difference or voltage drop ΔV across a device and the resulting current I that flows through the device.

$$\text{Resistance} = \frac{\text{potential difference}}{\text{current}}$$

$$R = \frac{\Delta V}{I}$$

The unit of electrical resistance is the **ohm** Ω (the Greek letter *omega*). From the definition, a device has a resistance of 1 ohm when a potential difference of 1 volt across the device causes a current of 1 ampere to flow through it.

$$1\ \Omega = 1\ \text{V/A}$$

Example 4.7 Resistance of a Lamp

The lamp in Figure 4.23 draws a current of 0.91 A when the power supply produces a potential difference of 110 V. What is the resistance of the lamp?

Solution: The potential difference, or voltage drop, across the lamp is the voltage of the power supply.

$$R = \frac{\Delta V}{I} = \frac{110\ \text{V}}{0.91\ \text{A}} = 121\ \text{V/A} \quad \text{or} \quad 121\ \Omega$$

The lamp's resistance is 121 ohms.

Ohm's Law

In 1826, the German physicist Georg Simon Ohm discovered that the ratio of potential difference to current is constant for most conductors. Thus, the resistance of most conductors does not vary as the magnitude or direction of the applied potential difference changes. Today we say that a device obeys **Ohm's law** if its resistance is constant, independent of potential difference and current.

$$\Delta V = IR$$

Although most conductors obey Ohm's law (at least for a limited range of voltage), many important devices do not. For example, Figure 4.24a on the next page shows the relationship between ΔV and I for a metallic conductor, where ΔV is directly proportional to I. What is the slope of the line in this graph? Is it constant? Figure 4.24b shows a similar graph for a semiconductor device where R is not constant—it decreases when V increases.

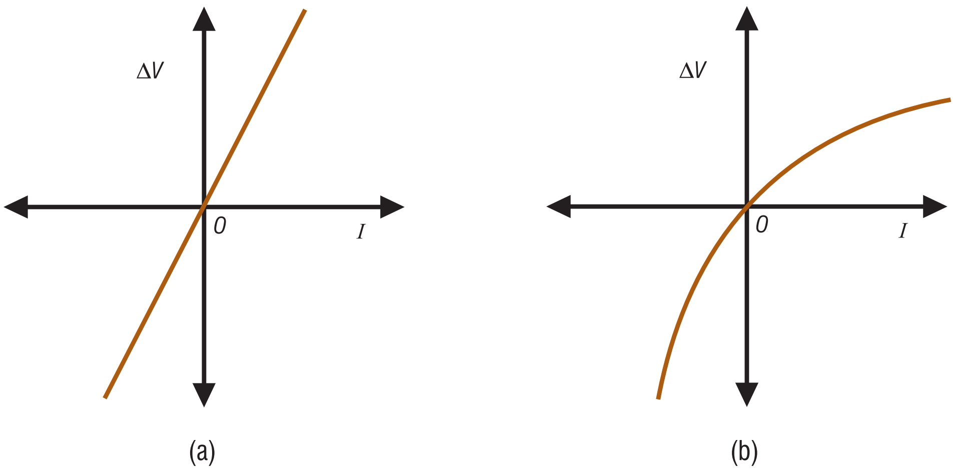
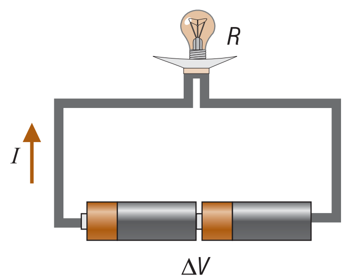


Figure 4.24
Graphs of the relationship between I and ΔV for
(a) a conductor that obeys Ohm's law, and
(b) a semiconductor device that does not obey Ohm's law.

Thus, Ohm's law is not a fundamental principle obeyed by all materials and devices. But the definition of resistance always applies to all devices. The resistance of a device is given by $R = \Delta V/I$ whether or not R is constant as the voltage changes.

Example 4.8 Ohm's Law in a Flashlight Bulb

New cells in a flashlight produce a total potential difference of 3.0 V. The flashlight bulb draws a current of 1.7 A with the new cells. As they are used, the cells degrade and their voltage output declines. If the bulb obeys Ohm's law, what current flows through the bulb when the cells produce a total potential difference of 2.5 V?



Solution: Use Ohm's law for the flashlight bulb. The resistance is the same for both potential differences. Let I represent the unknown current.

$$R = \frac{\Delta V}{I} = \frac{3.0\ \text{V}}{1.7\ \text{A}} = \frac{2.5\ \text{V}}{I}$$

$$I = \frac{(2.5\ \text{V})(1.7\ \text{A})}{3.0\ \text{V}} = 1.4\ \text{A}$$

When the potential difference is 2.5 V, the current flowing through the bulb is 1.4 A.

Resistivity

In Section 4.2, you learned that the amount of resistance to fluid flow through a pipe depends on three things:

- The length of the pipe—the longer the pipe, the greater the resistance.
- The radius (or cross-sectional area) of the pipe—the smaller the pipe, the greater the resistance.
- The material of which the fluid is composed—the higher the viscosity, the greater the resistance.

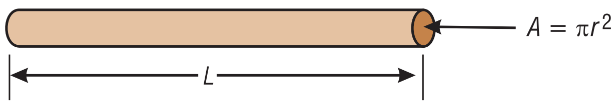


Figure 4.25
A longer, thinner pipe has a greater resistance to fluid flow. A longer, thinner wire has a greater resistance to charge flow.

Similarly, a wire has a resistance to charge flow that depends on:

- The length of the wire—the longer the wire, the greater the resistance.
- The radius (or cross-sectional area) of the wire—the smaller the wire, the greater the resistance.
- The material of which the wire is composed—the higher the *resistivity*, the greater the resistance.

Resistivity is a measure of the capacity of a material to resist electric charge flow. If a wire of unit cross-sectional area is made of the material, its resistivity is the resistance per unit length of the wire. In SI, the units of resistivity are $\Omega \cdot m$. The Greek letter ρ (rho) is used to represent resistivity. If the length L and cross-sectional area A of a wire are known, the wire's resistance R can be calculated:

$$R = \rho \frac{L}{A}$$

The resistivities of various materials are listed in Table 4.3.

Table 4.3 Approximate Resistivities (at 20°C)

Material	ρ ($\Omega \cdot$ m)	Material	ρ ($\Omega \cdot$ m)
Conductors		Semiconductors	
Copper	1.7×10^{-8}	Silicon	3.5×10^{-8}
Gold	2.4×10^{-8}	Carbon	1.4×10^{-5}
Aluminum	2.6×10^{-8}	Germanium	0.5
Iron	12×10^{-8}		
Mercury	98×10^{-8}	Insulators	
Nichrome Wire	112×10^{-8}	Glass	10^{12}
		Quartz	10^{17}

Example 4.9 Resistance of a Copper Wire

In the United States, wire is manufactured in standard sizes set by the American wire gage system. For example, No. 14 wire has a diameter of 1.63 mm. What is the resistance of a 100-m length of No.14 copper wire?

Solution: The radius of the wire is one-half the diameter. The cross-sectional area of the wire is

$$A = \pi r^2 = \pi \left(\frac{1.63 \text{ mm}}{2}\right)^2 = 2.09 \text{ mm}^2$$

Convert A to square meters. Use the conversion factor 1 m = 10^3 mm. Square both sides: $(1 \text{ m})^2 = (10^3 \text{ mm})^2 = 10^6 \text{ mm}^2$.

$$A = 2.09 \text{ mm}^2 \cdot \frac{1 \text{ m}^2}{10^6 \text{ mm}^2}$$

$$= 2.09 \times 10^{-6} \text{ m}^2$$

From Table 4.3, for copper at room temperature, $\rho = 1.7 \times 10^{-8}\ \Omega \cdot \text{m}$.

$$R = \rho \frac{L}{A} = (1.7 \times 10^{-8}\ \Omega \cdot \text{m}) \left(\frac{100 \text{ m}}{2.09 \times 10^{-6} \text{ m}^2}\right)$$

$$= 0.81\ \Omega$$

The resistance of 100 m of No. 14 copper wire is 0.81 Ω.

Example 4.10 Voltage Drop Across a Copper Wire

A 5.6-A current flows in the copper wire of Example 4.9. What is the potential difference between the ends of the wire?

Solution: Solve $R = \frac{\Delta V}{I}$ for ΔV:

$$\Delta V = RI$$

$$= (0.81\ \Omega)(5.6 \text{ A})$$

$$= 4.5 \text{ V}$$

The potential difference between the ends of the wire is 4.5 volts.

Series Circuits

Figure 4.26 shows a simple **series** circuit. Two lamps are connected in series with a power supply. When the switch is closed, there is only one path for current. Charge flows from the negative terminal of the power supply, through each lamp in turn, and into the positive terminal of the power supply.

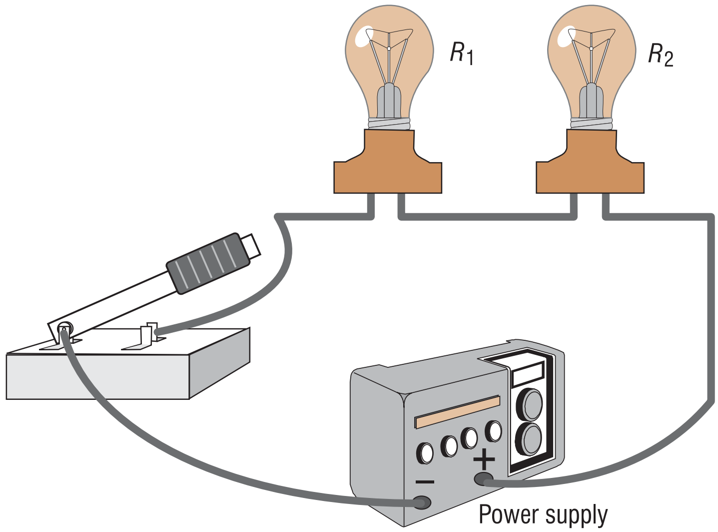


Figure 4.26
Two lamps connected in series with a power supply

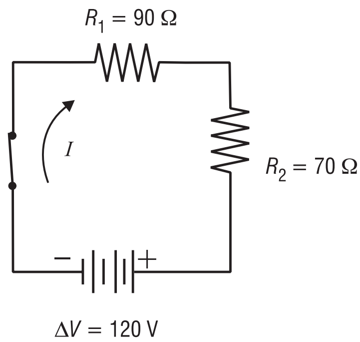
This simple circuit illustrates four important rules for series circuits:

1. Since the current has only one path, the current through each lamp, and everywhere in the series circuit, is the same.
2. The current is resisted by the first lamp, and then by the second lamp, in turn. The total resistance in a series circuit is the sum of the individual resistances along the path of the current.
3. From $\Delta V = RI$, the voltage drop across each lamp is the product of the lamp's resistance and the current.
4. The sum of the voltage drops across the lamps equals the total potential difference of the power supply. The total potential difference also equals the product of the total circuit resistance and the current.

Notice what happens when there is a break, or an *open circuit*, anywhere in the path of the current of a series circuit. For example, opening the switch or burning out one of the lamp filaments creates an open circuit in Figure 4.26. When this happens, charge flow stops at the break. As a result of rule 1, charge flow stops everywhere in the circuit. Thus, when electrical elements are connected in series, an open circuit stops the current everywhere.

Example 4.11 Resistance in Series

The lamps in Figure 4.26 have resistances of 90 Ω and 70 Ω. The power supply produces 120 V.



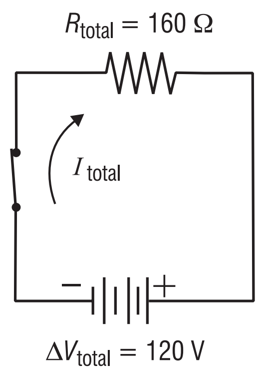
a. What is the current I through the circuit?

b. What is the voltage drop across each lamp?

Solution: First find the total resistance R_{total} of the circuit. Since the lamps are connected in series, R_{total} is the sum of the individual resistances (rule 2):

$$R_{total} = R_1 + R_2$$
$$= 90\ \Omega + 70\ \Omega = 160\ \Omega$$

a. To calculate the current, replace R_1 and R_2 by the single, equivalent resistance R_{total}. Use the definition of resistance.



$$R_{total} = \frac{\Delta V_{total}}{I_{total}}$$

$$I_{total} = \frac{\Delta V_{total}}{R_{total}} = \frac{120\text{ V}}{160\ \Omega} = 0.75\text{ A}$$

b. Let ΔV_1 be the voltage drop across R_1, and ΔV_2 be the voltage drop across R_2. Each voltage drop is the product of resistance and current (rule 3). The current through each resistance is the same (rule 1): $I_1 = I_2 = I_{total}$.

$$\Delta V_1 = R_1 I_1$$
$$= (90\ \Omega)(0.75\text{ A})$$
$$= 67.5\text{ V}$$

$$\Delta V_2 = R_2 I_2$$
$$= (70\ \Omega)(0.75\text{ A})$$
$$= 52.5\text{ V}$$

Check: The sum of the voltage drops should equal ΔV_{total}.

$$\Delta V_{total} \stackrel{?}{=} \Delta V_1 + \Delta V_2$$
$$120\text{ V} \stackrel{?}{=} 67.5\text{ V} + 52.5\text{ V}$$
$$120\text{ V} \stackrel{?}{=} 120\text{ V}\ \checkmark$$

Parallel Circuits

Figure 4.27 shows the two lamps connected in **parallel** with the power supply. When the switch is closed, part of the current from the power supply flows through one lamp and part flows through the other lamp.

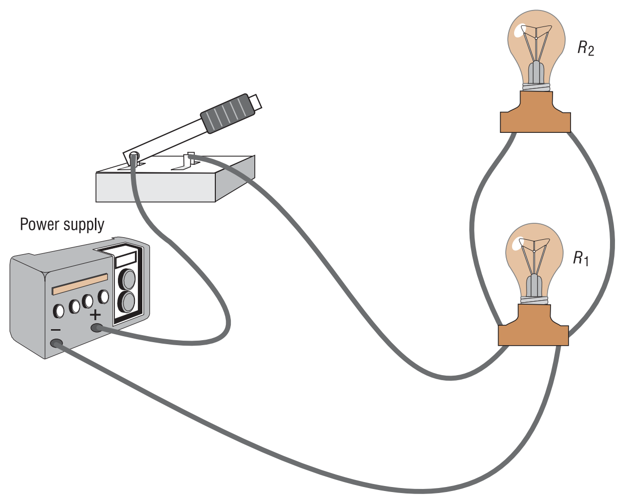


Figure 4.27
Two lamps connected in parallel with a power supply

This example illustrates four important rules for parallel circuits:

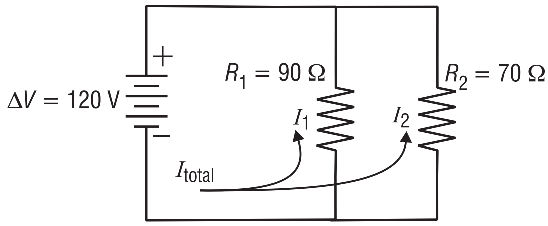
1. The total current through the circuit equals the sum of the currents through all the lamps.
2. Because there are multiple paths for current, the overall resistance of the circuit is decreased. The total resistance for the lamps connected in parallel is less than the resistance of either lamp. You can calculate the total resistance R_{total} using reciprocals: The reciprocal of R_{total} equals the sum of the reciprocals of the individual resistances.

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2}$$

3. From $I = \frac{\Delta V}{R}$, the current through each lamp is the ratio of the voltage drop across the lamp to the resistance of the lamp.
4. The voltage drop across the lamps is the same, and equals the potential difference of the power supply.

Example 4.12 Resistance in Parallel

The lamps in Figure 4.27 have resistances of 90 Ω and 70 Ω. The power supply produces 120 V.



a. What is the total current I_{total} in the circuit?

b. What is the voltage drop across each lamp?

c. What is the current in each lamp?

Solution: First find the total resistance R_{total} of the circuit. Since the lamps are connected in parallel, use reciprocals (rule 2):

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{total}} = \frac{1}{90\ \Omega} + \frac{1}{70\ \Omega}$$

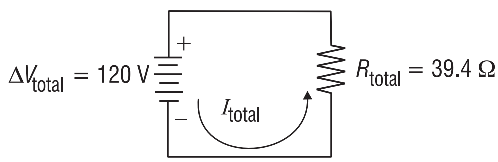
$$\frac{1}{R_{total}} = \frac{70\ \Omega + 90\ \Omega}{(90\ \Omega)(70\ \Omega)} = \frac{160\ \Omega}{6300\ \Omega^2}$$

To find R_{total}, find the reciprocal of $\frac{1}{R_{total}}$:

$$R_{total} = \frac{6300\ \Omega^2}{160\ \Omega}$$

$$= 39.4\ \Omega$$

a. To calculate the current, replace R_1 and R_2 by the single, equivalent resistance R_{total}.



$$R_{total} = \frac{\Delta V_{total}}{I_{total}}$$

$$I_{total} = \frac{\Delta V_{total}}{R_{total}} = \frac{120\ \text{V}}{39.4\ \Omega} = 3.05\ \text{A}$$

b. Let ΔV_1 be the voltage drop across R_1 and ΔV_2 be the voltage drop across R_2. The voltage drop across all lamps is the same, and equals the potential difference of the power supply (rule 4). Therefore,

$$\Delta V_1 = \Delta V_2 = \Delta V_{total} = 120 \text{ V}$$

c. Let I_1 be the current through R_1 and I_2 be the current through R_2.

$$I_1 = \frac{\Delta V_1}{R_1} = \frac{120 \text{ V}}{90\ \Omega} = 1.33 \text{ A}$$

$$I_2 = \frac{\Delta V_2}{R_2} = \frac{120 \text{ V}}{70\ \Omega} = 1.71 \text{ A}$$

Check: The sum of the currents should equal I_{total}.

$$I_{total} \stackrel{?}{=} I_1 + I_2$$

$$3.05 \text{ A} \stackrel{?}{=} 1.33 \text{ A} + 1.71 \text{ A}$$

$$3.05 \text{ A} \stackrel{?}{=} 3.04 \text{ A} \checkmark \quad \text{(within rounding error)}$$

Resistors

In the examples above, the lamps provide resistance in the circuits. The connecting wires also have resistance, but it is so small that we usually ignore it. Sometimes, additional resistance is needed in a circuit to reduce current. A **resistor** is an electrical device that has a specific resistance. Resistors are made of long wires, carbon, or semiconductors. The value of resistance is labeled on the outside of a resistor.

Suppose you needed to reduce the total current in the circuit of Example 4.12 to less than 2.0 A. The total resistance should be greater than 120 V/2.0 A or 60 Ω. The circuit already has a resistance of 39.4 Ω, so you need to add at least (60 Ω – 39.4 Ω) or 20.6 Ω. Therefore, you can reduce the current to less than 2.0 A if you add a 21-Ω resistor in series.

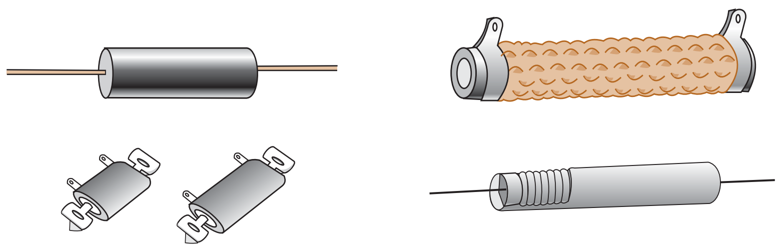


Figure 4.28
Resistor types

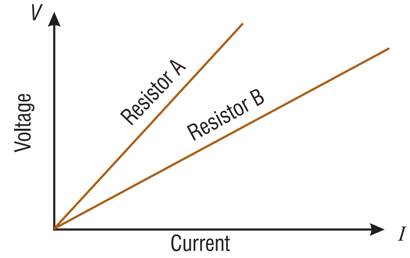
Summary

- Electrical resistance is the opposition to charge flow in electric circuits. The mathematical value of resistance is the ratio of potential difference across a device to current through the device. $R = \Delta V/I$. (Ω = V/A)
- If a material obeys Ohm's law, its resistance is constant, and $\Delta V = IR$.
- The resistance of a wire depends on the length and cross-sectional area of the wire and the resistivity of the material composition of the wire. $R = \rho L/A$.
- For resistors connected in series in a circuit, the total resistance is the sum of the individual resistances. The current is the same everywhere in the circuit. The sum of the voltage drops across the resistors equals the total voltage produced by the source.
- For resistors connected in parallel in a circuit, the reciprocal of the total resistance is the sum of the reciprocals of the individual resistances. The voltage drop is the same across each resistor. The total current in the circuit equals the sum of the currents through all resistors.

Exercises

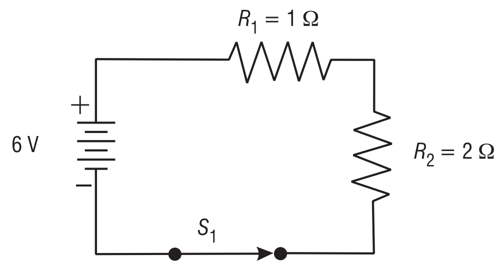
1. Electrical resistance in a wire is a measure of the opposition to the flow of ____________.
2. Copper wire and metallic solder are
   (a) conductors
   (b) semiconductors
   (c) insulators
3. The resistance of a wire *does not* depend on which of the following?
   (a) wire material
   (b) wire mass
   (c) wire cross-sectional area
   (d) wire length
4. The resistivity of a wire depends on
   (a) wire material
   (b) wire mass
   (c) wire cross-sectional area
   (d) wire length

5. The graph at the right shows the relationship between current and voltage for two resistors. Does resistor A or resistor B have the greater resistance?



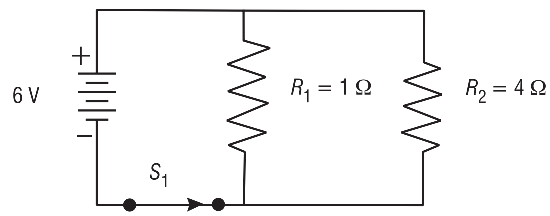
6. (a) What is the current through each resistor in the series circuit shown at the right?

   (b) What is the voltage drop across each resistor?



7. (a) What is the total current through the parallel circuit shown at the right?

   (b) What is the voltage drop across each resistor?



   (c) What is the current through each resistor?

8. (a) Sketch a circuit diagram showing three lamps connected in series with a power supply.

   (b) If one lamp burns out, what happens to the other two? Explain.

   (c) The intensity of light from a lamp depends on the current in the lamp filament. A high current causes a lamp to burn brightly, while a lower current causes dimming. If more lamps are connected in series to the circuit, what happens to the light intensity of each lamp?

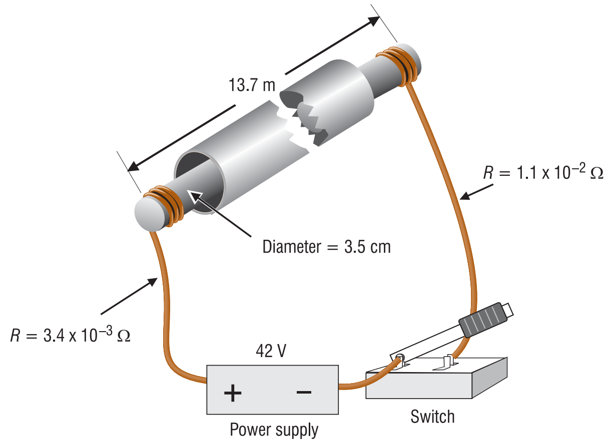
9. In the electrical circuit in your home, why are appliances, lighting, and outlets connected in parallel and not in series? Give two reasons.

10. Copper wires are available in a range of diameters. Number 12 wire is 2.05 mm in diameter, and number 00 wire is 9.27 mm in diameter. To carry a large current with a small drop in voltage, would you expect number 12 or number 00 wire to be used? Explain your answer.

11. The *mean free path* of electrons in a metal is the average distance traveled between collisions with atoms. The mean free path of electrons in metal A is greater than the mean free path in metal B. How would you expect the resistivity of metal A to compare to the resistivity of metal B?

12. Tell whether each of the following describes an open circuit or a closed circuit.
    (a) There is a break in the path of the current.
    (b) There is no charge flow in the circuit.
    (c) Current is measured in the circuit with an ammeter.
13. A circuit consists of a power supply and a single resistor.
    (a) What happens to the current through the resistor if the potential difference of the power supply doubles?
    (b) If the potential difference of the power supply does not change but a second identical resistor is connected in series, what happens to the current through the circuit?
14. What is the resistance of a car's headlamp if it draws a current of 2.5 amperes when connected to the car's 12-volt circuit?
15. An electric car has a 120-V power supply (ten 12-V batteries). The resistance of the motor is 0.21 Ω. What is the current through the motor?
16. (a) Draw a circuit diagram to show a 24-V battery and two resistors and an ammeter in series. The resistors have resistances of 12 Ω and 20 Ω.
    (b) What is the reading on the ammeter?
    (c) What is the voltage drop across each resistor?
17. (a) Draw a circuit diagram to show a 24-V battery and two resistors and a voltmeter in parallel. The resistors have resistances of 12 Ω and 20 Ω.
    (b) What is the reading on the voltmeter?
    (c) What is the current through each resistor?
18. Three 25-Ω resistors are connected in parallel and placed across a 60-V power supply.
    (a) What is the equivalent resistance of the parallel circuit?
    (b) What is the current through the power supply?
    (c) What is the current through each resistor?

19. As metal pipe is manufactured, it is inspected for flaws using a magnetic imaging process. The pipe is magnetized with current flowing through an aluminum rod inserted through the center of the pipe, as shown at the right. The rod has a diameter of 3.5 cm and is 13.7 m long.



Copper cables connect the rod to the terminals of a power supply. The resistance of each cable is shown above. When the switch is closed, a potential difference of 42 V is applied to the circuit.

(a) What is the resistance of the aluminum rod?

(b) What is the total resistance of the electrical circuit?

(c) Calculate the current in the circuit.

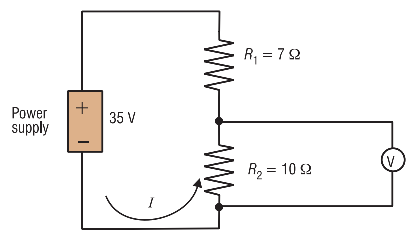
20. A portable electric heater draws a current of 10 amperes when it is connected directly to a household 120-volt outlet. The heater is connected to a 75-foot-long extension cord, which has a resistance of 1.8 ohms. The extension cord is plugged into a 120-volt outlet.

(a) What is the total resistance of the extension cord and heater?

(b) What current will flow through the circuit with the extension cord?

(c) What percentage of the heater's rated current of 10 A flows through the heater when it is connected through the extension cord?

21. Two resistors in series form a *voltage divider*, as shown at the right. Write your answers for (a) and (b) as fractions, not decimals.



(a) Calculate the total resistance R_{total} of the circuit. What is the ratio $\frac{R_2}{R_{total}}$?

(b) Calculate the voltage measured by the voltmeter across R_2. This is called the *output voltage* of the divider. The power supply voltage is called the *input voltage*. What is the ratio of output to input voltage?

(c) Compare your answers for (a) and (b). Explain how you could use a voltage divider to get an output voltage of x% of an input voltage.