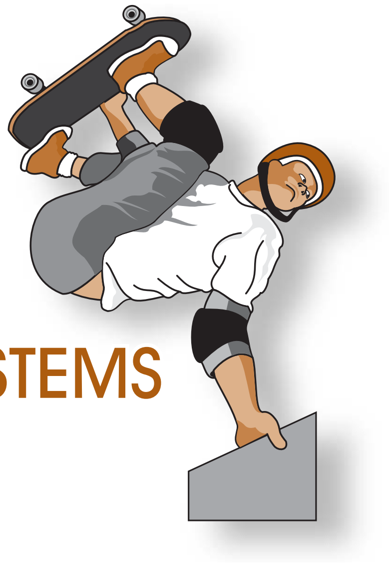
4.1 RESISTANCE IN MECHANICAL SYSTEMS



Objectives

- State Newton’s second law of motion and use it to solve problems involving force, mass, and acceleration.
- Calculate an object’s weight, given its mass.
- Explain the difference between static and kinetic friction.
- Use the linear model to calculate the force of friction between two surfaces.
- Explain how lubrication and rolling reduce friction.

In Section 1.1 you learned that the motion of a body in equilibrium is described by Newton’s first law of motion:

> *Every object will remain at rest, or will continue to move in a straight line with constant speed4 unless the object is acted on by a net force.*

A book sitting on a table is in equilibrium. The weight of the book is the force of gravity acting downward. The table pushes upward on the book with an equal and opposite force. Since the vector sum of these two forces is zero, no net force is acting on the book and it remains at rest.

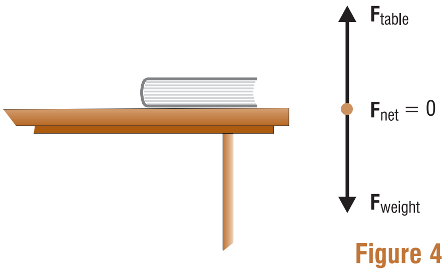


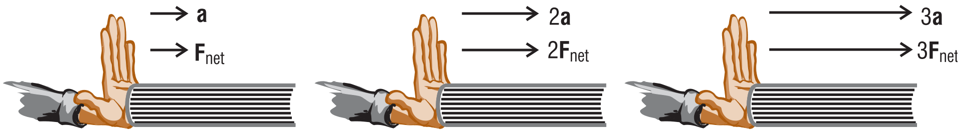
Figure 4.1
A book in equilibrium has no net force acting on it.

INTERNET connection

To find out more about resistance in mechanical systems, follow the links at **www.learningincontext.com.**

Newton's Second Law of Motion

Suppose you push the book and exert a net force in a horizontal direction from left to right. The book will begin to move, or be *accelerated*, in the direction of the net force. If you push with twice the force, the acceleration will double. If you push with three times the force, the acceleration will triple. In other words, the acceleration of the book is directly proportional to the force acting on the book.



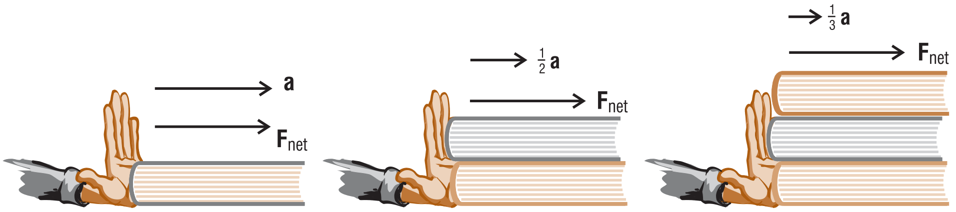
A force $\mathbf{F}_{\text{net}}$ results in an acceleration **a**.

Twice the force results in twice the acceleration.

Three times the force results in three times the acceleration.

Figure 4.2
Acceleration of an object is directly proportional to the force acting on the object.

Now suppose you use the same net force to accelerate more than one book. For an equal force, two books are accelerated at one-half the rate of one book. Three books are accelerated at one-third the rate, and so on. The acceleration of the books is inversely proportional to the mass.



A force $\mathbf{F}_{\text{net}}$ results in an acceleration **a**.

The force $\mathbf{F}_{\text{net}}$ results in half the acceleration for twice the mass.

The force $\mathbf{F}_{\text{net}}$ results in one-third the acceleration for three times the mass.

Figure 4.3
The acceleration of an object is indirectly proportional to the mass of the object.

The relationship between acceleration, net force, and mass is **Newton's second law of motion**:

> *The acceleration of an object is directly proportional to the net force acting on the object and inversely proportional to the mass of the object.*

Mathematically, Newton's second law is:

$$\text{Acceleration} = \frac{\text{net force}}{\text{mass}}$$

$$\boldsymbol{a} = \frac{\mathbf{F}_{\text{net}}}{m} \quad \text{or} \quad \mathbf{F}_{\text{net}} = m\boldsymbol{a}$$

In the SI system, force is measured in newtons (N). One newton is the force required to accelerate a 1-kg mass 1 m/s^2.

$$1\text{ N} = (1\text{ kg})\,(1\text{ m/s}^2) = 1\text{ kg}\cdot\text{m/s}^2$$

In the English system, force is measured in pounds (lb). One pound is the force required to accelerate a 1-slug mass 1 ft/s^2.

$$1\text{ lb} = (1\text{ slug})\,(1\text{ ft/s}^2) = 1\text{ slug}\cdot\text{ft/s}^2$$

Example 4.1 Newton's Second Law and a Car's Motion

In performance testing, a 1250-kg car accelerates from 0 to 100 km/h in 8.2 seconds. What is the average net force pushing the car during the test?

Solution: First convert the final speed to m/s:

$$v_f = \left(100\,\frac{\text{km}}{\text{h}}\right)\left(1000\,\frac{\text{m}}{\text{km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 27.8\,\frac{\text{m}}{\text{s}}$$

The car's average acceleration is the ratio of change in speed to the time interval:

$$a_{\text{ave}} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t}$$

$$= \frac{27.8\frac{\text{m}}{\text{s}} - 0\frac{\text{m}}{\text{s}}}{8.2\text{ s}}$$

$$= 3.39\text{ m/s}^2$$

Use Newton's second law to calculate the force.

$$F_{\text{ave}} = ma_{\text{ave}}$$

$$= (1250\text{ kg})\,(3.39\text{ m/s}^2)$$

$$= 4238\text{ N} \qquad \left[\text{N} = \text{kg}\cdot\text{m/s}^2\right]$$

The average net force pushing the car is 4240 N.

Calculating Weight and Mass

You can use Newton's second law to relate an object's weight to its mass. In this case, the acceleration is that experienced by the object in the Earth's gravitational field. We use the symbol $\mathbf{F}_g$ to represent gravitational force, or weight.

You have experienced gravitational acceleration if you have ever stepped off a diving board. While standing on the board, your weight $\mathbf{F}_g$ acting downward is balanced by an upward force exerted on you by the board. When you step off, the upward force is removed and a net force $\mathbf{F}_g$ is acting downward. You accelerate in the direction of this force, toward the surface of the pool.

When an object is in a gravitational field and gravity is the only force acting on the object, it accelerates in the direction of the force. This acceleration is called **gravitational acceleration**. We use the symbol $\mathbf{g}$ to represent gravitational acceleration. On the surface of the Earth, the direction of $\mathbf{g}$ is the same as $\mathbf{F}_g$, toward the center of the Earth and the magnitude is 9.80 m/s^2 or 32.2 ft/s^2.

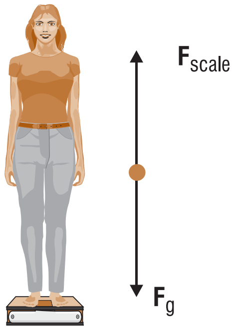
If an object's mass is *m*, you can use Newton's second law to calculate its weight:

$$\mathbf{F} = m\mathbf{a} \rightarrow \mathbf{F}_g = m\mathbf{g}$$

Example 4.2 Weight Measured by a Scale

A 65-kg person stands on a bathroom scale. What force does the scale exert on the person?

Solution: Two forces are acting on the person—the force of gravity, or the person's weight, F_g and the force of the scale F_{scale}. These two forces must be in equilibrium, since the person is standing still and is not being accelerated.



$$\begin{aligned} F_{scale} &= F_g = mg \\ &= (65\text{ kg})(9.80\text{ m/s}^2) \\ &= 637\text{ N} \qquad \left[\text{N} = \text{kg}\cdot\text{m/s}^2\right] \end{aligned}$$

The scale exerts a force of 637 N upward on the person.

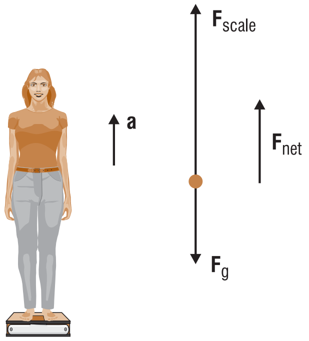
Notice in Example 4.2 that g is used to calculate force, even though nothing is being accelerated. If the force F_g is balanced by another force, there is no acceleration. But Newton's second law still holds, and you can calculate F_g using its equivalent, mg.

What does it feel like to be in an elevator when the elevator accelerates upward? You feel heavier, because the floor of the elevator is pushing upward with greater force than when you are standing still or moving at constant speed.

Example 4.3 Weight Measured During Acceleration

The person from Example 4.2 is placed on a scale in an elevator. The elevator accelerates upward at a rate of 2.5 m/s². What weight does the scale read during the acceleration?

Solution: Since the person is being accelerated upward, a net force $\mathbf{F}_{\text{net}}$ is acting upward. This force is the vector sum of the force of the scale (upward) and the force of gravity (downward). Let upward be the positive direction:



$$
\begin{aligned}
F_{\text{net}} &= F_{\text{scale}} - F_{\text{g}} \\
F_{\text{scale}} &= F_{\text{net}} + F_{\text{g}} \\
&= ma + mg \\
&= m(a + g) \\
&= (65\ \text{kg})\ (2.5\ \text{m/s}^2 + 9.80\ \text{m/s}^2) \\
&= 800\ \text{N}
\end{aligned}
$$

The scale reads 800 N during the acceleration.

Friction Forces

Refer to Appendix F for a career link to this concept.

When you push an object across a surface, such as the books in Figures 4.2 and 4.3 or the crate in Figure 4.4, across a surface, the force you apply to the object must be *greater than* the opposing force of **friction**. To accelerate an object that is standing still, you must overcome friction. To keep an object moving at constant speed, you also must apply a force to overcome friction. In either case, the force of friction resists motion.

Friction is a result of irregularities in the surfaces of objects.

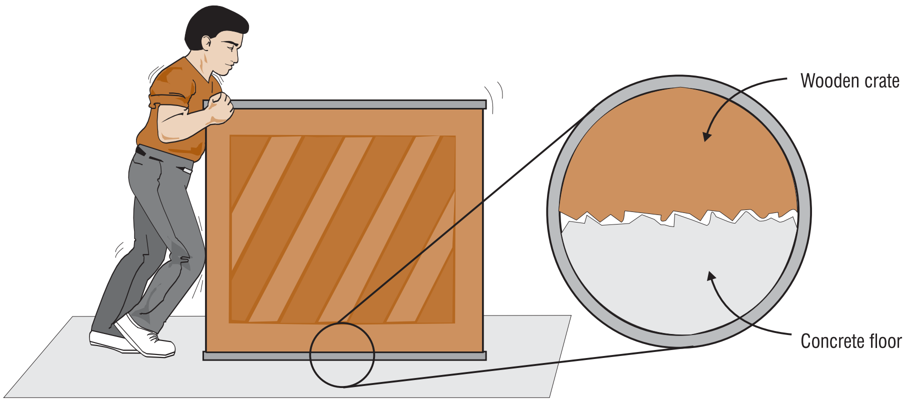


Figure 4.4
Magnified view of two surfaces in contact

At a microscopic level, all surfaces have irregular “hills and valleys.” The “hills” are the points of contact between surfaces. At these points, an electrical force of attraction, or bonding, occurs between atoms in the two surfaces. To begin sliding one surface over the other, you must break the bonds. The force required to overcome this initial attraction is called the **static friction** force. When the surfaces are moving, a weaker force of attraction still exists between atoms in the surfaces. You also must exert a force to overcome this attraction to maintain a constant speed. This force is called the **kinetic friction** force. Static friction is usually greater than kinetic friction.

A Linear Model for Friction Forces

Friction forces are very complicated, and no model can predict the forces accurately and reliably for all situations. However, a simplified linear model can be used to calculate the forces, which agree reasonably well with measured data.

The model is based on three observations about friction:

- The friction force depends on whether or not the surfaces are sliding (static or kinetic friction).
- The friction force depends on the materials of which the surfaces are made.
- The friction force depends on how hard the surfaces are pressed together. This is called the **normal force** between the surfaces. The direction of the normal force is perpendicular to the surfaces that are in contact.

Friction opposes motion. In other words, the direction of the friction force is opposite the direction of the applied force. According to the linear model, the magnitude of the friction force is proportional to the normal force N. The constant of proportionality is called the **coefficient of friction** μ (the Greek letter *mu*). The coefficient of static friction is μ_s and the coefficient of kinetic friction is μ_k.

$$\text{Maximum friction force} = \text{coefficient of friction} \times \text{normal force}$$

$$F_{\text{static}} \leq \mu_s N$$

$$F_{\text{kinetic}} \leq \mu_k N$$

Some typical values for μ_s and μ_k for various surface materials are shown in Table 4.1.

Table 4.1 Coefficients of Friction (Approximate Values)

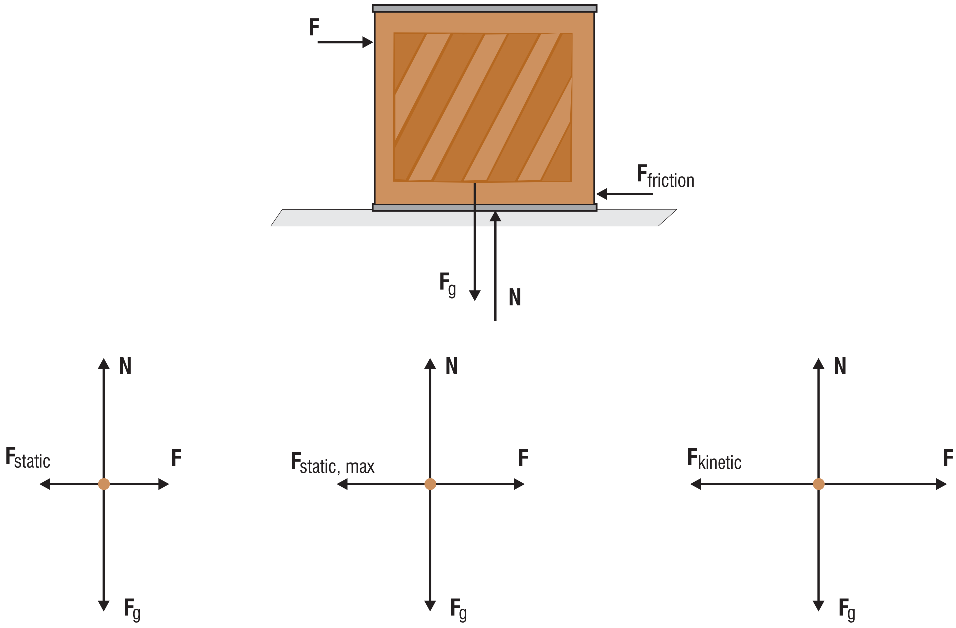
Surface	μ_s	μ_k
Wood on wood	0.5	0.2
Wood on concrete	0.6	0.4
Rubber on dry concrete	0.8	0.7
Rubber on wet concrete	0.65	0.57
Steel on steel (dry)	0.78	0.58
Steel on steel (oiled)	0.11	0.05
Steel on Teflon	0.04	0.04

To illustrate the forces in these equations, suppose you are pushing a crate with a force **F** directed to the right. You increase the magnitude of **F** until you get the crate moving (see Figure 4.5 on the next page):

(a) At first, $F < F_{\text{static,max}}$ and the crate does not move. Notice that $F_{\text{static,max}} = \mu_s N$. As you increase F, the force of friction F_{static} increases to exactly match F so that $F_{\text{static}} = F$.

(b) At the instant F becomes greater than $F_{\text{static,max}}$, a net force exists on the crate and it accelerates to the right. According to Newton's second law, the acceleration is proportional to the net force.

(c) With the crate moving, you must apply a force $F = F_{\text{kinetic}}$ to keep it moving at constant speed.



(a) $F < F_{\text{static,max}}$
$F = F_{\text{static}}$

(b) $F = F_{\text{static,max}}$

(c) $F = F_{\text{kinetic}}$

Figure 4.5
Forces acting on a crate

Example 4.4 Forces on a Crate

The crate in Figure 4.5 is made of wood and weighs 85 pounds. To slide the crate across a concrete floor:

(a) What force is required to get the crate moving?

(b) What force is required to keep the crate moving at constant speed?

Solution: (a) To get the crate moving, you apply a force **F** and gradually increase the magnitude of **F** until it just exceeds the opposing force $\mathbf{F}_{\text{static,max}}$. At the instant before it moves, the crate is in equilibrium (see Figure 4.5b). The vertical forces are balanced:

$$N = F_g = 85 \text{ lb}$$

The horizontal forces are also balanced. From Table 4.1, $\mu_s \approx 0.6$.

$$\begin{aligned} F = F_{\text{static,max}} &= \mu_s N \\ &= (0.6)\,(85 \text{ lb}) \\ &= 51 \text{ lb} \end{aligned}$$

To get the crate moving, the applied force must exceed 51 lb.

(b) When the crate moves at constant speed, there is no acceleration, and therefore no net force acting on the crate. In the vertical direction, the normal force still equals the weight of the crate. In the horizontal direction, the applied force now equals the force of kinetic friction (see Figure 4.5c). From Table 4.1, $\mu_k \approx 0.4$.

$$\begin{aligned} F = F_{\text{kinetic}} &= \mu_s N \\ &= (0.4)\,(85 \text{ lb}) \\ &= 34 \text{ lb} \end{aligned}$$

To keep the crate moving at constant speed, the applied force must equal 34 lb.

Lubricants

Lubricants reduce friction by keeping the two sliding surfaces apart with a thin layer of fluid. Figure 4.6 shows what happens when two surfaces slide with lubrication. The two objects—piston ring and cylinder wall—no longer touch each other.

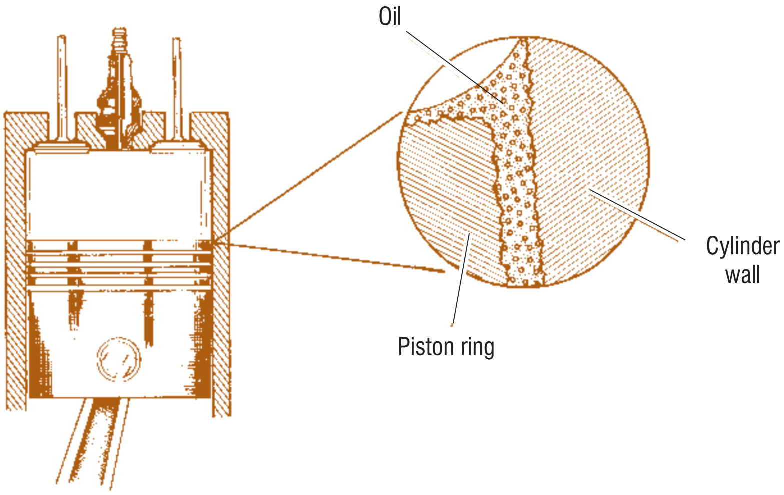


Figure 4.6
Effects of lubricants on friction

The lubricant between the surfaces moves in layers. The layer nearest the cylinder wall is stationary. Each layer farther away moves a little faster. The layer nearest the piston ring is moving as fast as the moving surface. Friction is no longer the result of tiny hills and valleys sliding past one another. Now friction is the result of fluid layers sliding past one another. This internal friction in the fluid is called *viscosity*. You will learn more about viscosity in the next section.

Proper lubrication reduces frictional forces between surfaces. This reduces the wear on the surfaces and the heat generated at the surfaces.

Rolling Friction

Another way to reduce friction is to roll an object over a surface, rather than slide it. Figure 4.7 shows a circular surface rolling on a flat surface. The surfaces don't scrape past one another as they do in sliding friction. This is why the force of rolling friction is always much less than that of sliding friction. An object fitted with wheels or other rolling surfaces moves with much less force than a similar object fitted with skids.

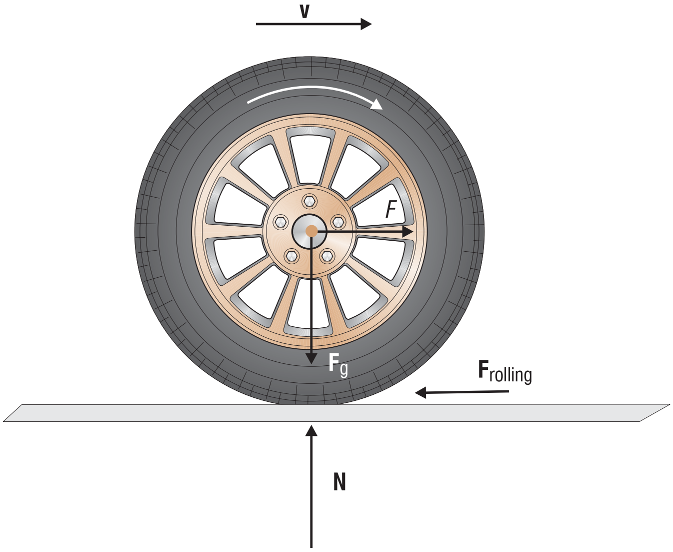


Figure 4.7
The force of rolling friction is much less than the force of sliding friction because there is less movement between surfaces.

Bearings are mechanical devices used to reduce friction in rotating systems. Sometimes bearings are lubricated to reduce friction even more.

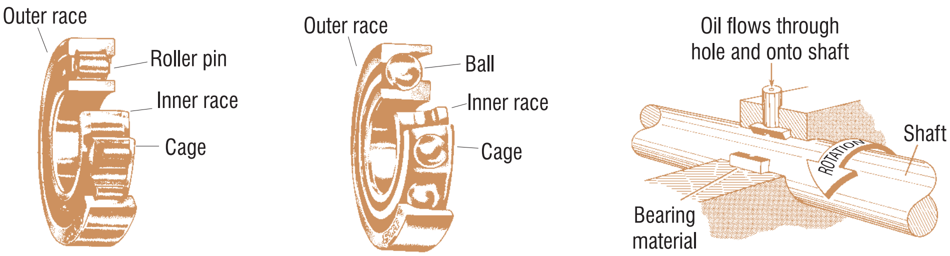


Figure 4.8
Bearings and lubrication reduce friction.

In some mechanical systems, friction is needed. For example, the brakes in your car use friction to slow the angular speed of the wheels and, in turn, the forward motion of the car. A friction clutch is used in some automobiles with manual-shift transmissions to transfer torque from the engine to the wheels. Friction is used in conveyer belt systems to move objects (see Figure 4.9). These systems work only if friction is present between the surfaces.

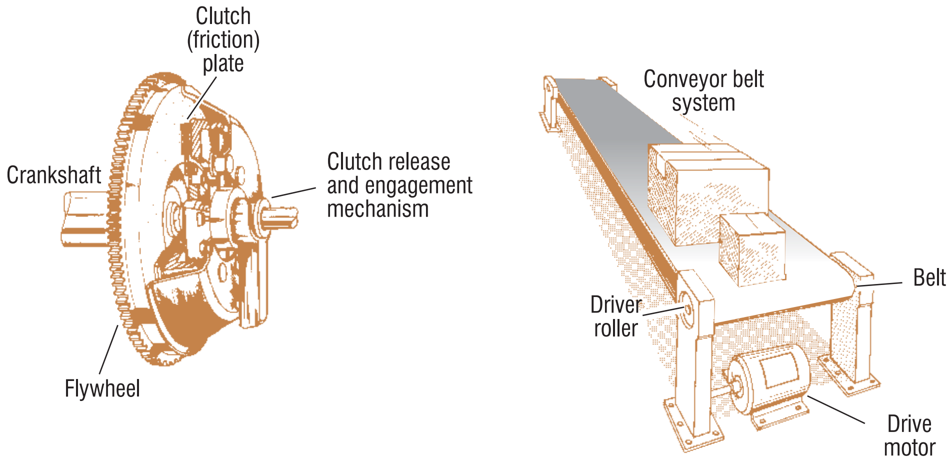


Figure 4.9
Mechanical systems that use friction

Summary

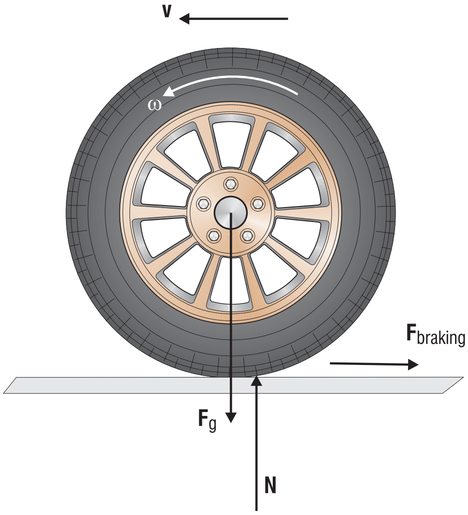
- Newton's second law of motion states the relationship among the net force acting on an object, the mass of the object, and its acceleration: $F = ma$.
- An object's weight is the force exerted on the object by gravity. $F_g = mg$. Near the surface of the Earth, the gravitational acceleration is $g = 9.80\ \text{m/s}^2$ or $32.2\ \text{ft/s}^2$.
- Friction causes resistance in mechanical systems.
- To start an object moving, you must overcome the force of static friction. To keep an object moving at constant speed, you must overcome the force of kinetic friction.
- According to the linear model, the force of friction between two surfaces is directly proportional to the normal force pushing the surfaces together.
- Lubrication and rollers reduce friction. Lubricants separate the two sliding surfaces with a layer of fluid.

Exercises

1. In Section 3.1 you learned that acceleration is the rate of change of velocity, $\frac{\Delta v}{\Delta t}$. In this section, you learned that acceleration is the ratio of force to mass, $\frac{F}{m}$. Are both these equalities true? Explain.
2. A small basket contains 1 kg of strawberries, and a large basket contains 2 kg. Does the large basket also contain twice the weight of the small basket? Explain.
3. Push a book slowly across a tabletop with your hand. Why is it harder to start the book sliding from a position of rest than it is to keep the book sliding when it is moving?
4. Suppose you drop an empty water bottle and an identical full water bottle from the same height. Which bottle will hit the ground first? Explain your answer.
5. Show that the units of g in the equation $g = F/m$ are m/s^2.
6. At liftoff, a 232,000-kg Delta II space launch vehicle is accelerated upward with a force exerted by its rocket engines. This force is called thrust. The Delta II rocket engines produce a thrust of 3.11 million newtons.
   (a) What is the net force acting on the Delta II at liftoff?
   (b) What is the acceleration of the Delta II at liftoff?
   (c) Following liftoff, the acceleration of the Delta II is not constant; it increases. Explain why this happens. (At liftoff, approximately 90% of the weight of the Delta II is rocket fuel.)
7. In hockey, a player takes a slap shot by striking the puck with a stick in an attempt to propel the puck into the opponent’s goal. Suppose that, in a slap shot, a force of 39 N causes a puck to accelerate at 230 m/s^2.
   (a) What is the mass of the hockey puck?
   (b) What acceleration would a 20-N force produce?
8. A 21-kg shipping box is at rest on a wooden floor. The coefficient of static friction between the box and the floor is 0.35, and the coefficient of kinetic friction is 0.28.
   (a) Find the magnitude of the horizontal force required to make the box start to slide.
   (b) Find the magnitude of the horizontal force required to keep the box sliding at a constant velocity.

9. An apple drops from a tree and falls straight down.
    (a) What is the acceleration at the instant the apple drops?
    (b) What is the velocity at the instant the apple drops?
    (c) What is the acceleration after 1 second of free fall?
    (d) What is the velocity after 1 second of free fall?

10. The illustration at the right shows a tire and a wheel on a car that is traveling to the left. When the driver applies the brakes, a force is exerted on the wheel at a distance from its axis of rotation. This force creates a torque that reduces the angular speed of the wheel. The reduction in angular speed slows the car—the car decelerates (or accelerates to the right). Thus, a braking force is applied to the car in the direction opposite its velocity



This braking force is exerted between tire and the road surface.

The braking force is the force of friction between the tire and the road. Suppose that, for a tire and a dry asphalt road, $\mu_s = 0.75$ and $\mu_k = 0.60$. The mass of the car and driver is 1460 kg.

(a) Calculate the normal force the road exerts on the tire. (Remember, the car has four tires.)

(b) What is the maximum braking force that can be exerted on one tire before the brakes “lock” and the tire starts to skid?

(c) What is the maximum braking force exerted on the tire if the brakes lock and the tire skids?

11. Would the maximum braking force from Exercise 10 increase or decrease under the following conditions? Explain your answers.
    (a) More people get into the car.
    (b) It starts raining.

12. A figure skater has a mass of 53 kg. The coefficient of kinetic friction between the ice and her skate is 0.15.
    (a) If the skater has only one skate touching the ice, what are the magnitude and direction of the normal force between the skate and ice?
    (b) What are the magnitude and direction of the kinetic friction force when the skater glides on one skate?
    (c) How much work must be done to move the skater 35 meters across the ice?
13. Your weight is the force exerted on you by gravity. In Newton's universal law of gravitation, this is the mutual force of attraction between you and the Earth. It is caused by your mass and the mass of the Earth. The force is given by the equation

$$F_g = G\frac{m_1 m_2}{d^2}$$

where $G = 6.67 \times 10^{-11}\ \frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}$. Let $m_1 = m_{\mathrm{Earth}} = 5.97 \times 10^{24}$ kg and $m_2 = m$ (your mass). Since you are on the surface of the Earth (or very near it), d is the distance from the center to the surface, or the radius of the Earth:

$$d = r_{\mathrm{Earth}} = 6.37 \times 10^6 \text{ m.}$$

In Newton's second law, your weight is the force of gravity that causes you to accelerate (in the absence of a balancing force) toward the center of the Earth. This force is the product of your mass and the acceleration of gravity.

$$F_g = mg$$

Your weight is the same for both interpretations of force:

$$mg = G\frac{m_{\mathrm{Earth}}\, m}{F_{\mathrm{Earth}}{}^2}$$

Solve this equation for g, and compare your numerical result to the standard value. Does g depend on your mass?