3.4 Rate in Thermal Systems

Objectives

- Define heat flow rate and its SI and English units of measure.
- Describe the heat transfer processes of conduction, convection, and radiation.
- Solve heat transfer rate problems using the heat conduction equation.

Heat Flow Rate

Rate in mechanical, fluid, and electrical systems is the movement of some substance—something that has mass and volume, like automobiles, water, and electrons. Rate in thermal systems is the movement of heat. Heat is not a substance. You learned in Section 1.4 that heat is thermal energy transferred from one body to another because of a temperature difference. The amount of heat that is transferred per unit time is the **heat flow rate**.

INTERNET connection

To find out more about rate in thermal systems, follow the links at **www.learningincontext.com.**

If an amount of heat Q is transferred from one body to another in a time interval Δt, the heat flow rate is given by the following ratio

$$\text{Heat flow rate} = \frac{\text{heat transferred}}{\text{time interval}}$$

$$\dot{Q} = \frac{Q}{\Delta t}$$

There are several possible combinations for the units of heat flow rate. Heat transferred can be measured in any energy unit: calories (cal), joules (J), British thermal units (Btu), or foot pounds (ft·lb). The units for time interval can be seconds (s), minutes (min), or hours (h). Therefore, in SI, heat flow rate can be measured in cal/s, cal/min, cal/h, J/s, J/min, or J/h. In the English system, heat flow rate can be measured in Btu/s, Btu/min, Btu/h, ft·lb/s, ft·lb/min, or ft·lb/h.

Table 3.2 gives some useful conversion factors for converting between energy units.

Table 3.2 Energy Conversion Factors

1 cal	=	4.185 J
1 cal	=	3.077 ft·lb
1 Btu	=	1054 J
1 Btu	=	252 cal
1 Btu	=	778 ft·lb
1 ft·lb	=	1.356 J

Example 3.13 Heat Flow Rate in Air Conditioning

Devices are classified by their heat flow rate capacities using the English units "Btu" for "Btu per hour." For example, an air-conditioning unit classified as 15,000 Btu is capable of removing heat from a building at a rate of 15,000 Btu/hr. If this unit runs continuously for 3.5 hours, what amount of heat is transferred? Write the answer in Btu and J.

Solution: $\dot{Q} = \dfrac{Q}{\Delta t}$

$$Q = \dot{Q}\Delta t$$

$$= \left(15{,}000\ \frac{\text{Btu}}{\text{h}}\right)(3.5\ \text{h})$$

$$= 52{,}500\ \text{Btu or } (52{,}500\ \text{Btu})\left(\frac{1054\ \text{J}}{1\ \text{Btu}}\right) = 5.53 \times 10^{7}\ \text{J}$$

The air-conditioning unit transfers 52,500 Btu or 5.53×10^7 J of heat in 3.5 hours.

Heat Conduction

When you place a pot on a hot stove, heat is transferred from the stove's heating element to the bottom of the pot. The atoms and molecules in the bottom of the pot vibrate faster and faster as the temperature increases. When these energetic particles collide with particles in the side of the pot, they transfer some of their energy. As the atoms and molecules in the side of the pot gain thermal energy, the temperature in the side increases. These particles, in succession, collide with other particles and transfer energy farther up the side of the pot. This process continues until there is no temperature difference on the pot. The transfer of thermal energy arising from a temperature difference between adjacent parts of a body is called **heat conduction**.

Will heat be conducted in an object if the entire object is at the same temperature? In this case, all the molecules in the object have the same average kinetic energy. Half the molecules have energy greater than the average, and half have energy less than the average. A molecule has the same chance of gaining energy in a collision as it does of losing energy. Therefore, there is no net flow of thermal energy in the object. For heat conduction to take place, there must be a temperature difference within an object.

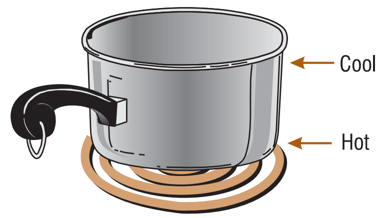


Figure 3.20
Heat conduction is the transfer of thermal energy within the same body caused by a temperature difference.

Not all substances have the same ability to conduct heat. For example, the molecules of air and other gases are so far apart that their collision rates are low. Therefore, gases are poor conductors of heat. Molecules of liquids and some nonmetallic solids are closer together, and these materials are better conductors of heat than gases. Metals have the greatest ability to conduct heat. In a metal, many electrons are not tightly bound to atoms and are free to move about the metal. These free electrons are responsible for a metal's ability to conduct heat as well as electrical current. After a collision, a free electron can move past many atoms before colliding and giving up energy. This increase in distance of travel of energetic particles speeds the transfer of energy from high-temperature regions to low-temperature regions.

The **thermal conductivity** k of a material is a measure of its ability to conduct heat. Metals have a large value of k because they are good heat conductors. Wood has a small value of k because it is a poor heat conductor. Wood is a good thermal insulator. This is why wood makes a good handle for a pot.

The rate at which heat is conducted through a slab of material (see Figure 3.21) depends on the thermal conductivity k of the material, the temperature difference ΔT across the slab, the cross-sectional area A through which heat flows, and the thickness Δx of the slab.

$$\text{Heat conduction rate} = -\frac{\begin{matrix}\text{thermal}\\\text{conductivity}\end{matrix} \times \begin{matrix}\text{cross-sectional}\\\text{area}\end{matrix} \times \begin{matrix}\text{temperature}\\\text{difference}\end{matrix}}{\text{thickness}}$$

$$\dot{Q} = -\frac{kA\Delta T}{\Delta x}$$

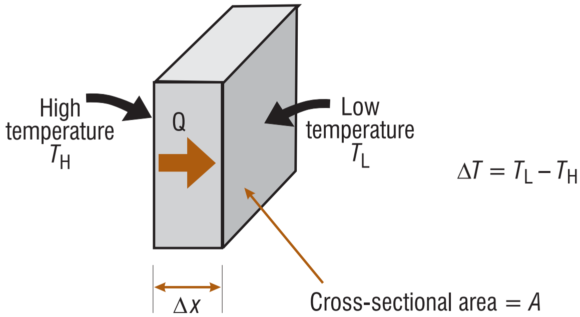


Figure 3.21
The rate of heat conduction $\dot{Q}$ through a slab is equal to $-kA\Delta T/\Delta x$.
A "slab" of material has a small thickness Δx compared to the height and width.

We use the same variable name $\dot{Q}$ for heat transfer rate and heat conduction rate. This is because heat conduction is a special case of heat transfer—a case where heat is transferred within the same body. Therefore, the units of heat conduction rate are the same as those for heat transfer rate.

Notice the minus sign in the equation for heat conduction rate. By convention, ΔT means "final temperature minus initial temperature." In the direction of heat conduction the final temperature is T_L and the initial temperature is T_H. The temperature difference $\Delta T = T_L - T_H$ is negative since $T_H > T_L$. Therefore, the heat conduction rate $\dot{Q}$ is positive. Heat flows from a region of high temperature to a region of low temperature.

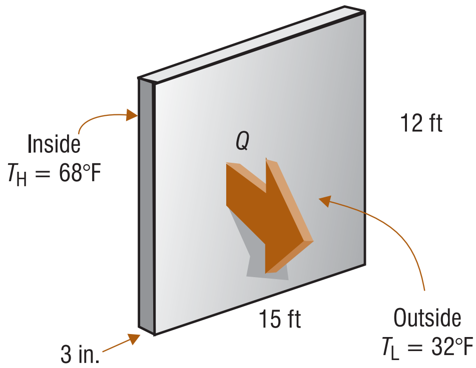
Table 3.3 gives the thermal conductivity (k) for several substances. Note that metals have higher thermal conductivities than insulating materials such as air or cork board.

Table 3.3 Some Thermal Conductivities

Material	$\left(\frac{\text{Btu} \cdot \text{in}}{\text{h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}}\right)$	$\left(\frac{\text{cal} \cdot \text{cm}}{\text{s} \cdot \text{cm}^3 \cdot {}^\circ\text{C}}\right)$
Polyurethane Foam	0.17	5.7×10^{-5}
Air	0.18	6.3×10^{-5}
Fiberglass	0.32	1.1×10^{-4}
White Pine	0.78	2.69×10^{-4}
Water	4.25	1.46×10^{-3}
Glass	5.8	1.99×10^{-3}
Concrete	6.0	2.07×10^{-3}
Steel	350	0.12
Aluminum	1400	0.48
Copper	2700	0.93

Example 3.14 Heat Flow Through Fiberglass

A wall of a house is designed to contain a 3-in. thickness of fiberglass insulation. The wall measures 15 ft by 12 ft. What is the heat flow rate through the insulation if the inside temperature is 68°F and the outside temperature is 32°F?



Solution: The cross-sectional area of the wall is

$$A = (15 \text{ ft})\,(12 \text{ ft}) = 180 \text{ ft}^2$$

The temperature difference is

$$\Delta T = T_L - T_H = 32°\text{F} - 68°\text{F} = -36°\text{F}$$

The heat transfer rate is

$$\dot{Q} = -kA\,\frac{\Delta T}{\Delta x}$$

$$= -\left(0.32\,\frac{\text{Btu}\cdot\text{in}}{\text{h}\cdot\text{ft}^2\cdot°\text{F}}\right)(180 \text{ ft}^2)\left(\frac{-36°\text{F}}{3 \text{ in}}\right)$$

$$= 691\,\frac{\text{Btu}}{\text{h}}$$

Heat flows from the inside of the insulation (high temperature) to the outside (low temperature) at a rate of 691 Btu/h.

If the wall in Example 3.14 is insulated with polyurethane foam instead of fiberglass, will the heat transfer rate increase, decrease, or stay the same?

In addition to conduction, there are two other methods of transferring heat from regions of higher temperature to regions of lower temperature: convection and radiation.

Refer to Appendix F for a career link to this concept.

Convection

Convection is a transfer of heat by movement of fluid. The process is complex and cannot be described by one simple equation. Figure 3.22 shows two bodies with different temperatures, not in contact with each other, but both in contact with a fluid. At the surface of the high-temperature body, fluid gains energy by conduction. This causes the fluid near the surface to expand (the distance between molecules increases when they gain energy). When the fluid expands, it becomes less dense than the surrounding cooler fluid and rises because of the buoyant forces acting on it. When the warm fluid rises, it is replaced by cool fluid. This fluid will, in turn, gain energy from the high-temperature body and rise.

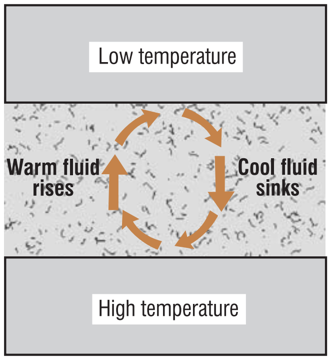


Figure 3.22
Natural convection

The result is a circulation of fluid and transfer of energy due to a temperature difference. Energy is transferred from a high-temperature body to the fluid; the warmed fluid rises and transfers energy to the cooler fluid and low-temperature body. This kind of heat transfer is called *natural convection*. Natural convection takes place in the Earth's atmosphere. The sun heats the surface of the Earth, energy is transferred from the surface to the air, and the air rises and cools at higher altitudes. Since different parts of the Earth's surface absorb heat from the sun more rapidly than others, the air near the surface is heated unevenly. This is what produces convection currents, or winds, in the atmosphere.

Convection can also be forced. In *forced convection*, a fan or pump creates a pressure difference in a heated or cooled fluid, which forces the fluid to circulate. This is how heating and cooling systems work. For example, the human body uses blood as the working fluid in a complex forced convection system for cooling. Thermal energy is released by metabolism inside the body (for example, in muscles) and is transferred to blood. Blood is forced to circulate by the heart (a pump). The warmed blood is pumped to the skin, where thermal energy passes from the blood to the skin. The energy is removed from the skin by convection, by a change of phase (evaporation of moisture from the skin), and by another method called **radiation**.

Radiation

All objects radiate energy in the form of *electromagnetic radiation*. This is the same type of radiation as that emitted by a light bulb, but most electromagnetic radiation is not visible—it has a wavelength shorter or longer than visible light. (You will learn about the characteristics of electromagnetic radiation in Chapter 9.) The rate of energy radiated by an object depends on the object's temperature, surface area, and material composition of the surface. The rate of energy emission increases with the fourth power of the temperature. Radiation transfers energy from one body to another through empty space—it does not use a medium.

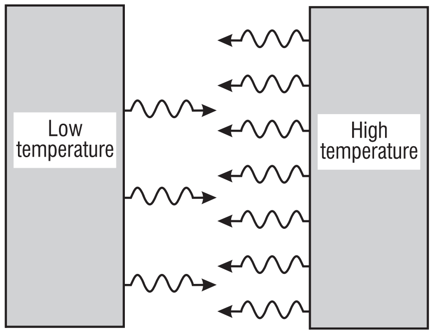


Figure 3.23
A hot object radiates more energy per unit time than an identical cooler object.

In Figure 3.23, a hot object radiates energy, some of which is absorbed by a cooler object. The cooler object also radiates energy, but, because it has a lower temperature, it radiates less than it absorbs. The net transfer of energy is from the hot object to the cooler object. Radiation is the most common means of energy transfer in the universe. Energy is transferred from the sun to all the planets in the solar system by radiation.

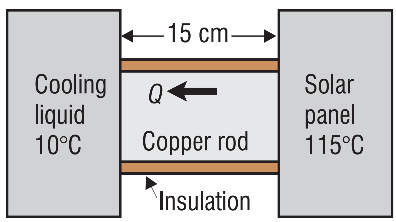
Summary

- Heat flow rate is the amount of thermal energy transferred per unit time.
- Heat is transferred from a high-temperature object or region to a low-temperature object or region.
- Heat conduction is the transfer of thermal energy within an object due to a temperature difference between adjacent regions of the object.
- The thermal conductivity of a material is a measure of its ability to conduct heat.

- Convection is a transfer of heat by movement of fluid. Convection can be natural or forced.
- Radiation is a transfer of energy by electromagnetic waves.

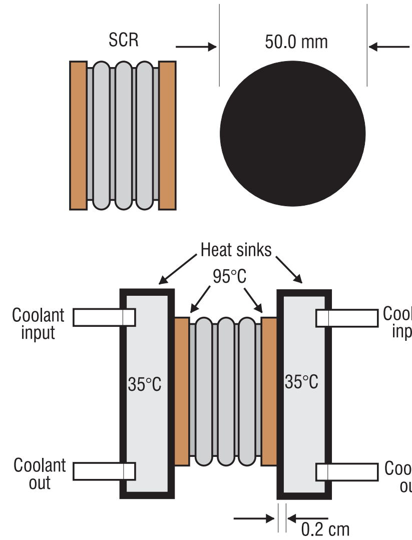
Exercises

1. What condition is necessary for heat to flow from one region to another region of the same object?
2. Suppose you leave a metal spoon and a wooden spoon in a refrigerator overnight. The next day you remove the spoons and hold one in each hand. The metal spoon feels colder than the wooden spoon, even though they are the same temperature.
   (a) Referring to Table 3.4, which spoon is a better conductor?
   (b) Explain why the metal spoon feels colder.
3. In choosing a material for a cooking utensil, would you want the material to have a high or low thermal conductivity? Explain your answer.
4. A pine wood door 1.75 in. thick will be removed and replaced by glass. How thick should the glass be if the heat flow rate through the wall is to be unchanged?
5. In an electric water heater, the heating element is located at the bottom of the tank. Why is this advantageous?
6. Why does warm fluid rise in natural convection?
7. The Earth continuously absorbs radiation from the sun. Why doesn't the temperature of the Earth continuously increase?
8. When a lake begins to freeze, ice forms first at the surface. Is conduction, convection, or radiation involved in this process? Explain your answer.
9. A solid copper rod is used as a "heat pipe" to conduct heat from a hot solar panel to a cooling liquid. The cross-sectional area of the rod is 1.2 cm^2 and the length is 15 cm. The rod is insulated around the side, so the only heat conduction is through the ends of the rod. The solar panel temperature is 115°C and the cooling liquid temperature is 10°C. What is the heat flow rate through the rod?



10. The heat flow rate from the solar panel in Exercise 9 must be increased to 10.0 cal/s. If you can change only the temperature of the cooling liquid, what new temperature should you use?

11. A silicon control rectifier (SCR) is a semiconductor device used to control large AC power devices. During operation, current flows through the SCR and heats it. The added thermal energy must be removed or it will destroy the SCR.



As shown at the right, an SCR is clamped between two "heat sinks" that contain a low-temperature coolant. The ends of the SCR are separated from the coolant by 0.20 cm of steel. The temperature of the SCR is 95°C, and the temperature of the coolant is 35°C. What is the heat flow rate, in J/s, from each end of the SCR? You can ignore heat flow from the sides of the SCR.

12. A 50-gram sample of water is initially at a temperature of 22°C. The sample is heated until the temperature is 32°C. The specific heat of water is $1.00 \ \frac{\text{cal}}{\text{g} \cdot {}^\circ\text{C}}$.

(a) How much heat is absorbed by the water, in calories?

(b) in Btu?

(c) in joules?

(d) in ft•lb?

13. Three liters of water evaporate from cooking pots in a commercial kitchen. The water vapor mixes with air in the kitchen, and the mixture passes through the kitchen's air conditioner. In the air conditioner, the mixture is cooled and water vapor condenses back into liquid water. *When water vapor condenses, it releases heat. The amount of heat released is equal to the amount of heat absorbed when the water was evaporated.* The heat released in condensation must be removed by the air conditioner. The air conditioner is rated at 12,000 Btu per hour.

(a) The heat of vaporization of water is 540 cal/g. How much heat is absorbed by the water when 3 L are evaporated? (The density of water is 1 g/cm^3, and there are 1000 cm^3 in 1 L.)

(b) How long will it take for the air conditioner to remove the heat released by the condensing water?