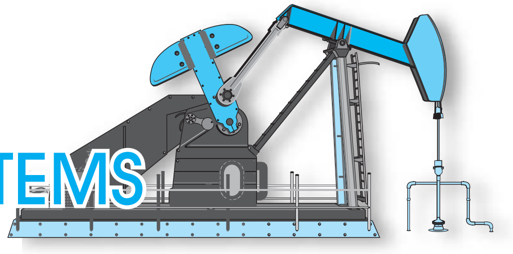
3.2 Rate in Fluid Systems



Objectives

- Define volume flow rate.
- Solve problems using the volume flow rate equation.
- Define mass flow rate.
- Solve problems using the mass flow rate equation.
- Explain how volume and mass flow rates can be measured.

Rates in fluid systems describe the motion of liquids and gases. Fluids in motion have speed and acceleration. However, we use two different rates to describe how quickly fluids flow—*volume flow rate* and *mass flow rate*.

For example, fans move air to control temperatures inside buildings, to cool electronic components in computer systems, and to force air into combustion chambers in jet engines. In all of these fluid systems, the amount of air moved by the fan over a period of time can be specified as a fluid flow rate.

INTERNET connection

To find out more about rate in fluid systems, follow the links at **www.learningincontext.com.**

Volume Flow Rate

Let V represent the volume of fluid in a container. Suppose fluid flows out of the container, so that V changes in an interval of time. We use ΔV to represent the change in volume, or the volume of fluid moved in a time interval Δt. The **volume flow rate** is the change in volume per unit time, or the ratio $\frac{\Delta V}{\Delta t}$. We use a dot over the variable to represent *rate of change*. Thus, $\dot{V}$ represents volume flow rate.

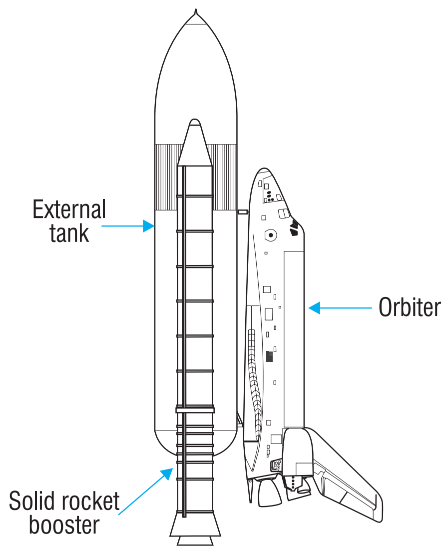
$$\text{Volume flow rate} = \frac{\text{change in volume}}{\text{time interval}}$$

$$\dot{V} = \frac{\Delta V}{\Delta t}$$

Volume can be measured in many units, for example m^3, cm^3, ft^3, in^3, liters, or gallons. Thus, volume flow rate can be measured in m^3/s, ft^3/min, liters/h, etc.

Example 3.7 Volume Flow Rate of Liquid Oxygen

The space shuttle's main engine burns a mixture of hydrogen and oxygen as its fuel. The shuttle's external fuel tank stores liquid hydrogen and liquid oxygen in separate tanks. The liquid oxygen tank holds 143,000 gallons. During liftoff, the flow rate of liquid oxygen to the shuttle's main engine is 16,800 gallons per minute. At this rate, how long can liquid oxygen be supplied to the main engine?



The space shuttle external tank contains a liquid oxygen tank and a liquid hydrogen tank.

Solution: The volume of fluid moved is 143,000 gal, and the flow rate is 16,800 gal/min. Solve the volume flow rate equation for the time interval.

$$\dot{V} = \frac{\Delta V}{\Delta t}$$

$$\Delta t = \frac{\Delta V}{\dot{V}} = \frac{143{,}000 \text{ gal}}{16{,}800 \ \dfrac{\text{gal}}{\text{min}}} = 8.51 \ \text{min}$$

Liquid oxygen can be supplied to the main engine for 8.51 minutes.

Mass Flow Rate

The **mass flow rate** is the mass of fluid moved per unit time. We use the symbol m to represent mass and $\dot{m}$ to represent mass flow rate. If Δm is the change in mass, or the mass of fluid moved in a time interval Δt, the mass flow rate is

$$\text{Mass flow rate} = \frac{\text{change in mass}}{\text{time interval}}$$

$$\dot{m} = \frac{\Delta m}{\Delta t}$$

Example 3.8 Mass Flow Rate of Liquid Oxygen

The space shuttle's external fuel tank holds 226,000 pounds of liquid hydrogen. If all the fuel in the external tank is consumed during the first 8.5 minutes of flight, what is the mass flow rate of liquid hydrogen, in slugs per second?

Solution: A mass of 1 slug weighs 32.2 pounds. Convert the weight to mass:

$$\Delta m = 226{,}000 \text{ lb} \cdot \frac{1 \text{ slug}}{32.2 \text{ lb}} = 7019 \text{ slugs}$$

$$\dot{m} = \frac{\Delta m}{\Delta t} = \frac{7019 \text{ slugs}}{8.5 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ s}} = 13.8 \text{ slugs/s}$$

The mass flow rate of liquid hydrogen is 13.8 slugs/s.

Refer to Appendix F for a career link to this concept.

Measuring Fluid Rates

Methods for measuring volume and mass flow rates vary, depending on the situation. The simplest, most direct method to find average flow rate is to collect fluid and measure the amount of fluid collected over a time interval. The volume or mass of fluid collected divided by the time interval is the average flow rate. *Flowmeters* use indirect methods that do not significantly interrupt the flow. These instruments typically measure fluid properties such as pressure and velocity. From these values, the instruments can calculate volume or mass flow rate.

For example, suppose an instrument measures the speed of a fluid flowing through a pipe. The measurement is taken over a time interval Δt. This interval is small enough that the fluid speed does not change during the measurement. Now imagine a particle suspended in the fluid, and moving with the fluid at the fluid speed. How far does this particle move in the time interval Δt? As shown in Figure 3.11, if the fluid speed is v, it moves a distance $L = v\Delta t$.

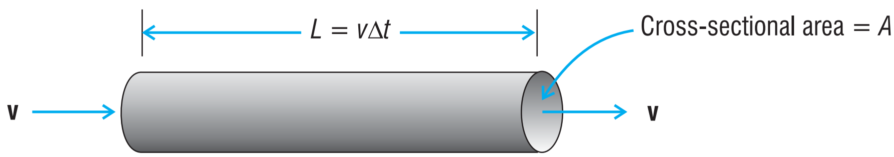


Figure 3.11
A particle moving at constant speed v moves a distance $v\Delta t$ in a time interval Δt.

What volume of fluid enters this section of pipe from the left during the time interval Δt? As long as this fluid completely fills the pipe, the volume of the entering fluid must equal the volume of the section of pipe shown. Let A represent the cross-sectional area of the pipe. The volume of the pipe section is the product of the length L and cross-sectional area A. Thus, the volume of fluid ΔV moving through the pipe section in time interval Δt is:

$$\begin{aligned}\Delta V &= AL \\ &= Av\Delta t\end{aligned}$$

Divide both sides of this equation by Δt:

$$\frac{\Delta V}{\Delta t} = \frac{Av\Delta t}{\Delta t} = Av$$

The ratio $\frac{\Delta V}{\Delta t}$ is the volume flow rate ˙. We have shown that, if an instrument measures a fluid's speed, it can calculate the volume flow rate by multiplying the speed by the cross-sectional area of the flow.

$$\text{Volume flow rate} = \text{cross-sectional area} \times \text{fluid speed}$$

$$\dot{V} = Av$$

To find the mass flow rate, you can use this equation and the density of the fluid. Density is the ratio of mass to volume. If the density ρ of the fluid does not change as it flows through the section of pipe,

$$\text{Density} = \frac{\text{mass moved}}{\text{volume moved}}$$

$$\rho = \frac{\Delta m}{\Delta V}$$

Solve for Δm:

$$\Delta m = \rho \Delta V$$

Divide both sides of this equation by the time interval Δt:

$$\frac{\Delta m}{\Delta t} = \rho \frac{\Delta V}{\Delta t}$$

$$\dot{m} = \rho \dot{V}$$

Since $\dot{V} = Av$, substitute:

$$\dot{m} = \rho Av$$

If an instrument measures a fluid's speed, it can calculate the mass flow rate by multiplying three terms:

$$\text{Mass flow rate} = \text{fluid density} \times \text{cross-sectional area} \times \text{fluid speed}$$

$$\dot{m} = \rho Av$$

Example 3.9 Speed of Liquid Oxygen

During liftoff, the space shuttle's liquid oxygen flows from the storage tank to the main engine. If it flows through a discharge pipe of diameter 17 inches, what is the speed of the liquid oxygen in the pipe in feet per second?

Solution: From Example 3.7, the volume flow rate is 16,800 gallons per minute. There are 231 in^3 per gallon, so

$$\dot{V} = 16{,}800\ \frac{\text{gal}}{\text{min}} \cdot 231\frac{\text{in}^3}{\text{gal}} \cdot \frac{1\ \text{min}}{60\ \text{s}} = 64{,}680\ \text{in}^3/\text{s}$$

The radius of the pipe is 8.5 inches. The cross-sectional area is

$$A = \pi r^2 = \pi(8.5\ \text{in})^2 = 227\ \text{in}^2$$

Solve the volume flow rate equation for v.

$$\dot{V} = Av$$

$$v = \frac{\dot{V}}{A}$$

$$= \frac{64{,}680\ \frac{\text{in}^3}{\text{s}}}{227\ \text{in}^2} = 285\ \text{in/s}$$

Convert to feet per second.

$$v = 285\frac{\text{in}}{\text{s}} \cdot \frac{1\ \text{ft}}{12\ \text{in}} = 23.75\ \text{ft/s}$$

The speed of the liquid oxygen in the discharge pipe is 23.75 feet per second.

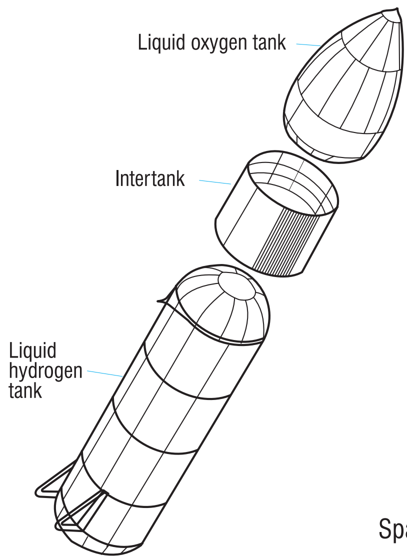


Figure 3.12
Space shuttle external tank components

Example 3.10 Pipe Diameter for Liquid Hydrogen Flow

You are designing a discharge pipe for the space shuttle's liquid hydrogen tank. The design engineer wants the hydrogen fluid speed to be eight times the oxygen speed. What diameter pipe should you use in the design? The weight density of liquid hydrogen is 4.42 lb/ft³.

Solution: Convert weight density to mass density:

$$\rho = 4.42\ \frac{\text{lb}}{\text{ft}^3} \cdot \frac{1\ \text{slug}}{32.2\ \text{lb}} = 0.137\ \frac{\text{slug}}{\text{ft}^3}$$

The liquid hydrogen speed is eight times the liquid oxygen speed, found in Example 3.9.

$$v = 8 \times 23.75\ \text{ft/s} = 190\ \text{ft/s}$$

From Example 3.8, the mass flow rate of liquid hydrogen is 13.8 slugs/s. Use the mass flow rate equation, and solve for the cross-sectional area.

$$\dot{m} = \rho A v$$

$$A = \frac{\dot{m}}{\rho v}$$

$$= \frac{13.8\frac{\text{slugs}}{\text{s}}}{0.137\frac{\text{slug}}{\text{ft}^3} \cdot 190\frac{\text{ft}}{\text{s}}} = 0.530\ \text{ft}^2$$

The pipe is circular, so its cross-sectional area is

$$A = \pi r^2$$

$$r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{0.530\ \text{ft}^2}{\pi}} = 0.41\ \text{ft}$$

Convert to inches:

$$r = 0.41\ \text{ft} \cdot \frac{12\ \text{in}}{\text{ft}} = 4.9\ \text{in}$$

You should use a 4.9-in.-radius, or 9.8-in.-diameter pipe for the liquid hydrogen discharge.

Processes That Use Rates of Fluids

Fluid systems that measure and control flow rates include bottling plants for products such as soft drinks, dishwashing liquid, lubricating oil, and bottled water. In these uses, a pump moves the fluid from a mixing tank to a bottling machine. The rate of fluid flow is set so that the bottles are filled quickly.

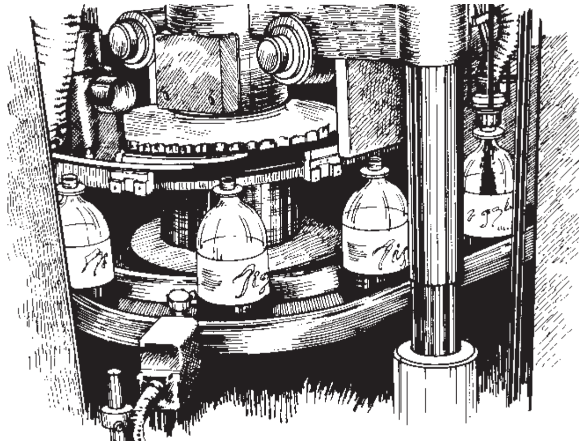


Figure 3.13
Bottling machine

City water supply systems generally use large-diameter pipes to carry water to homes and business places. In fluid systems that use large-diameter pipes, the speed of the fluid is slow. But the volume flow rate is high. Using smaller pipes and higher flow speeds might also meet the demand. But the work required to move the same amount of water through smaller pipes would be higher and, thus, less efficient.

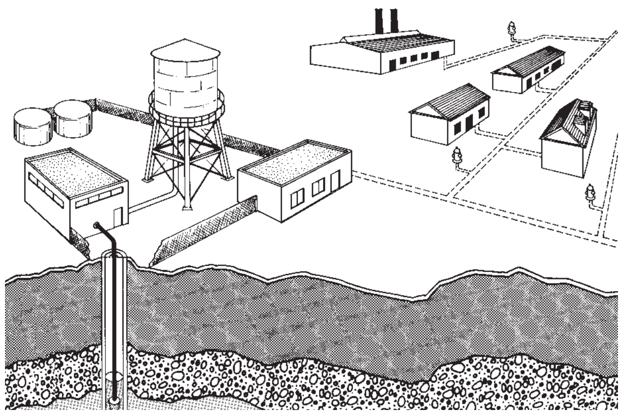


Figure 3.14
A city water supply system

In the case of gases (especially air, which is in common use), it's necessary to put the storage tank as close to the point of use as possible. The flow rate of compressed air through a hose or pipe is reduced when the air has to travel a long distance rather than a short distance. This reduced flow rate is caused by frictional losses between the moving air and the inside surface of the hose or pipe. (You'll learn more about friction in Chapter 4.) Locating the compressor and storage tank close to the point of use makes it easier to maintain the required volume flow rate of a system.

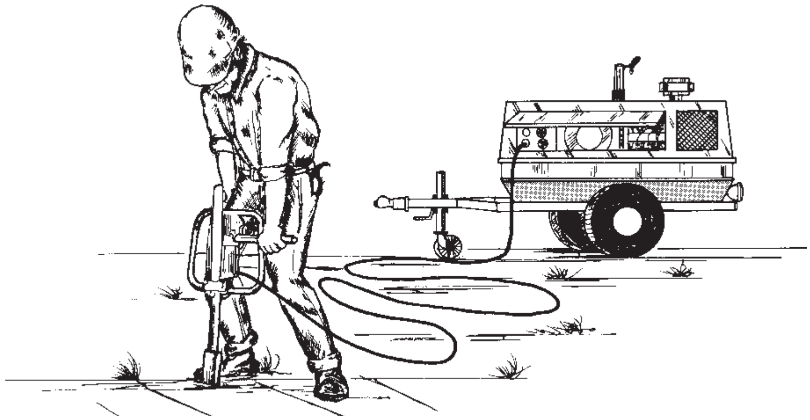


Figure 3.15
Control of flow rate from air compressor to jackhammer is important.

Summary

- Volume flow rate is the volume of fluid moved divided by the time interval over which it is moved.
- Typical volume flow rates are measured in gal/min, ft^3/s, liters/s, or m^3/s.
- Mass flow rate is the mass of fluid moved divided by the time interval over which it is moved.
- Typical mass flow rates are slugs/s and kg/s.

Exercises

The following relations will be useful when you need to make unit conversions.

$$1 \text{ gal} = 231 \text{ in}^3 = 0.1337 \text{ ft}^3 = 3.785 \times 10^{-3} \text{ m}^3$$

1. Label the following quantities as volume flow rate, mass flow rate, or neither.

   (a) 12 slugs/min (b) 0.4 cm^3/min (c) 50 g/s (d) 7.8 L/h
   (e) 6.9 kg/h (f) 4.2 m^2/s (g) 0.8 gal/s (h) 2.5 gal/min

2. Some people develop partial blockages of blood flow due to deposits of plaque on the inside walls of their arteries. Suppose a blockage reduces the diameter of an artery by one-half but the speed of blood flow does not change. Does the blood volume flow rate decrease by one-half? Explain your answer.

3. The compressor of a car's air-conditioning system pumps 5 kg of refrigerant in 3 minutes. What is the mass flow rate through the system in kg/min?

4. The pump on a truck used to fight brush fires empties the 1200-gallon tank on the truck in 4.8 minutes. What is the flow rate through the pump in gpm (gallons/minute)?

5. A large diesel truck has a speed of 60 mph. The fuel consumption of the truck at this speed is 7 miles/gallon.

   (a) What is the flow rate, in gpm, through the fuel injection pump on the engine?

   (b) What is the flow rate expressed as in^3/s?

6. The pump for a water-cooling system delivers 18 gpm of water through a pipe with an inside diameter of 1.00 inch.

   (a) What is the speed of water through the pipe in ft/min?

   (b) If the pressure change in the pipe is –23 psi, how much work, in ft·lb, is done in one second to move the water through the pipe?

7. The thrust of a rocket engine, F_{thrust}, can be calculated with mass flow rate. Let v represent the speed of the exhaust gas in m/s, and let $\dot{m}$ represent the "burn rate" or the rate at which fuel is used. $\dot{m}$ is the mass flow rate in kg/s. $\dot{m}$ will always be a negative number since the rocket is using fuel, not making fuel.

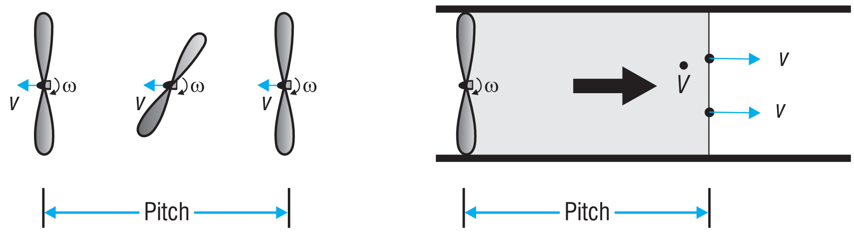
$$F_{thrust} = -v\dot{m}$$

   (a) What are the units for thrust using this equation?

   (b) A rocket engine has an exhaust velocity of 12,000 m/s and burns 580 kg of fuel in 2 minutes. Find the thrust of the rocket engine.

8. Mass flow is the critical factor in many industrial chemical reactions. Hydrogen gas feeding a hydrogenation reaction has a density of 1.24×10^{-3} g/cm^3 and a speed of 75.3 cm/s through a pipe with a diameter of 1.27 cm. What is the mass flow rate $\dot{m}$ of hydrogen to the reaction?

9. When a boat's propeller rotates, it moves forward in the water in the same way (but less efficiently) that a screw moves forward when it rotates in a block of wood. The propeller's *pitch* is the horizontal distance the propeller moves forward in one complete revolution, without the influence of drag. For example, a propeller with a 6-in. pitch moves forward 6 inches per revolution.



A propeller can also be used in a flowmeter. Water moving past the propeller forces it to rotate. A propeller with a 6-in. pitch will rotate one complete revolution when a column of water 6 inches long flows past the propeller.

(a) Water flows through a pipe containing a flowmeter with a 6-in.-pitch propeller. The propeller turns at 90 rpm. What is the speed, in inches per second, of the water?

(b) The diameter of the pipe is 4.5 inches. What is the pipe's cross-sectional area?

(c) Calculate the volume flow rate of the water, in cubic inches per second.

(d) Calculate the volume flow rate in gallons per minute.