3.1 Rate in Mechanical Systems

Objectives

- Define speed, velocity, and acceleration.
- Explain the difference between speed and velocity.
- Explain the difference between velocity and acceleration.
- Use speed, velocity, and acceleration to solve problems involving linear motion.
- Define angular speed and angular acceleration.
- Use angular speed and angular acceleration to solve problems involving rotational motion.

In mechanical systems, rates describe how quickly something changes. Speed or velocity describes how quickly the position of an object changes. Acceleration describes how quickly velocity changes. You can use speed, velocity, and acceleration to analyze the motion of objects moving in a straight line, a circle, a curved path, or back-and-forth vibration.

INTERNET connection

To find out more about rate in mechanical systems, follow the links at **www.learningincontext.com.**

Speed

If an object is in motion, it travels some distance in a given time interval. The **speed** of the object is the ratio of the distance traveled to the time interval. You can measure speed with a tape measure and stopwatch.

For example, to determine the speed of an automobile, you use a tape measure and stopwatch to find the distance traveled in some interval of time. Suppose you observe the automobile to travel 5 meters in 1 second, 10 meters in 2 seconds, 15 meters in 3 seconds, and so on for five seconds. The observed times t and distances d are shown in Table 3.1. The data are plotted on a coordinate plane in Figure 3.1.

Table 3.1
Time and Distance Data

Time, t	Distance, d
1 s	5 m
2 s	10 m
3 s	15 m
4 s	20 m
5 s	25 m

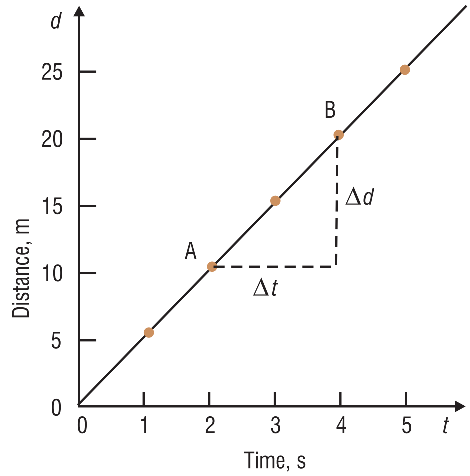


Figure 3.1
Time and distance plot for a car traveling at constant speed

Notice the data fall on a straight line. The **slope** of the line, the ratio of the *rise* to the *run*, is constant—it is the same everywhere on the line. If you locate any two points A and B on the line, the rise is the distance Δd traveled between the points and the run is the time interval Δt. Therefore, the speed equals the slope:

$$\text{Speed} = \frac{\text{distance traveled}}{\text{time interval}} = \frac{\Delta d}{\Delta t}$$

For points A and B in Figure 3.1, let the (t, d) coordinates be A (t_1, d_1) and B (t_2, d_2). The speed is calculated as follows:

$$\text{Speed} = \frac{\Delta d}{\Delta t} = \frac{d_2 - d_1}{t_2 - d_1} = \frac{20\text{ m} - 10\text{ m}}{4\text{ s} - 2\text{ s}}$$

$$\text{Speed} = \frac{10\text{ m}}{2\text{ s}}$$

$$= 5\text{ m/s} \quad \text{or} \quad 5\text{ meters per second}$$

Select any two points (t_1, d_1) and (t_2, d_2) from Table 3.1. Calculate the speed between these points. Is the speed 5 m/s?

Now suppose the measured time and distance data at A and B are the same, but the speed of the automobile between A and B varies. Sample measurements of time and distance between A and B are plotted in Figure 3.2.

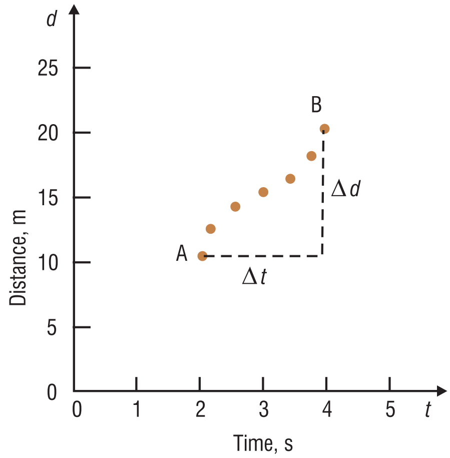


Figure 3.2
Time and distance plot for a car traveling at a varying speed

You can still calculate the speed between A and B as $\Delta d/\Delta t$, but this ratio is now more precisely called the **average speed** of the automobile between A and B. We use the symbol v for speed and v_{ave} for average speed.

$$\text{Average speed} = \frac{\text{distance traveled}}{\text{time interval}}$$

$$v_{ave} = \frac{\Delta d}{\Delta t}$$

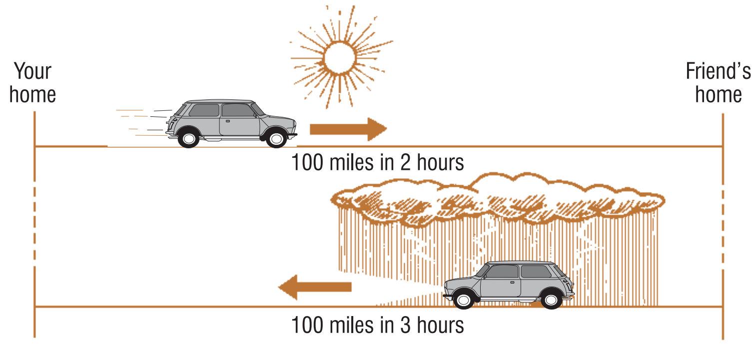
The speed v at any one instant is called the **instantaneous speed**. This is the speed indicated by the automobile's speedometer. As shown in Figure 3.1, if the instantaneous speed is constant, $v = v_{ave}$. But, as shown in Figure 3.2, the instantaneous speed is often quite different from the average speed.

You can calculate the average speed between A and B as follows: Let the (t, d) coordinates of A and B be $A(t_1, d_1)$ and $B(t_2, d_2)$.

$$\begin{aligned} v_{ave} &= \frac{\Delta d}{\Delta t} = \frac{d_2 - d_1}{t_2 - t_1} \\ &= \frac{20\text{ m} - 10\text{ m}}{4\text{ s} - 2\text{ s}} \\ &= \frac{10\text{ m}}{2\text{ s}} \\ &= 5\text{ m/s} \end{aligned}$$

Example 3.1 Average Speeds in a Round Trip

You leave home to visit a friend and drive 100 miles east. The trip takes two hours. On the return trip home, you drive through a rainstorm, and take three hours, driving 100 miles west, to return. Calculate (a) the average speed driving east to your friend's home, (b) the average speed driving west to your home, and (c) the average speed for the total round trip.



Solution: (a) Driving east to your friend's home:

$$v_{\text{ave}} = \frac{\Delta d}{\Delta t} = \frac{100 \text{ mi}}{2 \text{ h}} = 50 \text{ mi/h} \quad \text{or} \quad 50 \text{ mph}$$

(b) Driving west to your home:

$$v_{\text{ave}} = \frac{\Delta d}{\Delta t} = \frac{100 \text{ mi}}{3 \text{ h}} = 33.3 \text{ mi/h} \quad \text{or} \quad 33.3 \text{ mph}$$

(c) Driving the round trip:

$$v_{\text{ave}} = \frac{\Delta d}{\Delta t} = \frac{200 \text{ mi}}{5 \text{ h}} = 40 \text{ mi/h} \quad \text{or} \quad 40 \text{ mph}$$

During the trip to your friend's home, your average speed is 50 mph. On the return trip, you average 33.3 mph. For the total round trip, your average speed is 40 mph. Notice that the average for the round trip does not equal the average of the eastbound and westbound averages. Can you explain why?

Velocity

In Example 3.1, the speed and direction of two trips were important—50 mph east and 33.3 mph west. When we state the speed and direction of an object's motion, we are stating the object's **velocity**. Velocity **v** is a vector quantity. (Remember, we use boldface to represent vector quantities.) Speed v is the magnitude of the object's velocity. Speed is a scalar quantity.

If an object moves with a constant velocity, it moves with constant speed along a straight-line path. But constant speed and constant velocity are not the same thing. A race car can travel around a circular race track at constant speed, but its velocity is continuously changing because its direction is continuously changing.

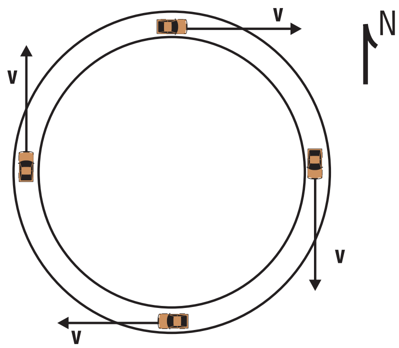


Figure 3.3
On a circular race track, a car's speed can be constant, but its velocity changes continuously.

You can calculate velocity, a vector, as a ratio. But instead of a scalar value for distance traveled, you need to use *displacement*.

Thus,

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{Velocity} = \frac{\text{displacement}}{\text{time}}$$

Displacement is a vector that defines the distance and direction between two positions. For example, Cincinnati is 145 km northeast of Louisville. If you fly from Louisville to Cincinnati, your displacement is 145 km northeast. The magnitude of the displacement vector is 145 km—the distance traveled. The direction is northeast. We use the symbol $\Delta\mathbf{d}$ for displacement.

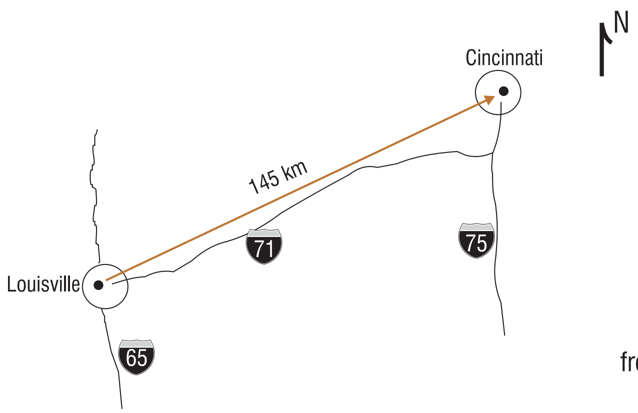


Figure 3.4
The displacement vector from Louisville to Cincinnati is 145 km northeast.

$$\text{Average velocity} = \frac{\text{displacement}}{\text{time interval}}$$

$$\mathbf{v}_{ave} = \frac{\Delta \mathbf{d}}{\Delta t}$$

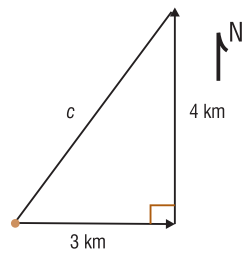
What is your average velocity if your flight from Louisville to Cincinnati takes 0.8 hour?

Example 3.2 Average Velocity is Not Average Speed

A bicyclist travels east for 3 kilometers in 5 minutes, then turns north and travels 4 kilometers in 5 minutes. Is the magnitude of the average velocity the same as the average speed for each leg of the trip? For the total trip?

Solution: The bicycle trip has two legs: One has a displacement of 3 km east, and one has a displacement of 4 km north. The net displacement is the vector sum. Since the two legs are perpendicular, the three vectors form a right triangle. The magnitude of the net displacement can be found using the Pythagorean theorem:

$$\begin{aligned} c^2 &= 3^2 + 4^2 \\ &= 9 + 16 \\ &= 25 \\ c &= 5 \text{ km} \end{aligned}$$



For the eastbound leg, the displacement is 3 km east:

$$v_{\text{ave}} = \frac{\Delta d}{\Delta t} = \frac{3 \text{ km}}{5 \text{ min}} \cdot 60 \frac{\text{min}}{\text{h}} = 36 \text{ km/h}$$

$$\mathbf{v}_{\text{ave}} = \frac{\Delta \mathbf{d}}{\Delta t} = \frac{3 \text{ km east}}{5 \text{ min}} \cdot 60 \frac{\text{min}}{\text{h}} = 36 \text{ km/h east}$$

v_{ave} = magnitude of $\mathbf{v}_{\text{ave}}$ for the eastbound leg

For the northbound leg, the displacement is 4 km north:

$$v_{\text{ave}} = \frac{\Delta d}{\Delta t} = \frac{4 \text{ km}}{5 \text{ min}} \cdot 60 \frac{\text{min}}{\text{h}} = 48 \text{ km/h}$$

$$\mathbf{v}_{\text{ave}} = \frac{\Delta \mathbf{d}}{\Delta t} = \frac{4 \text{ km north}}{5 \text{ min}} \cdot 60 \frac{\text{min}}{\text{h}} = 48 \text{ km/h north}$$

v_{ave} = magnitude of $\mathbf{v}_{\text{ave}}$ for the northbound leg

For the total trip, the distance traveled is 3 km + 4 km = 7 km, and the time interval is 5 min + 5 min = 10 min. The displacement is 5 km northeast:

$$v_{\text{ave}} = \frac{\Delta d}{\Delta t} = \frac{7\text{ km}}{10\text{ min}} \cdot 60\frac{\text{min}}{\text{h}} = 42\text{ km/h}$$

$$\mathbf{v}_{\text{ave}} = \frac{\Delta \mathbf{d}}{\Delta t} = \frac{5\text{ km northeast}}{10\text{ min}} \cdot 60\frac{\text{min}}{\text{h}} = 30\text{ km/h northeast}$$

$$v_{\text{ave}} \neq \text{magnitude of } \mathbf{v}_{\text{ave}} \text{ for the total trip.}$$

Example 3.2 illustrates a general property of speed and velocity: When an object travels from one point to another, the average speed depends on the path traveled but the average velocity does not. This is because you use the actual distance traveled to calculate average speed but you use the displacement vector to calculate average velocity. The displacement vector depends on only the object's initial and final locations, not the path traveled.

Acceleration

Velocity describes the motion of an object as the rate of change of its position. **Acceleration** describes the rate of change of an object's velocity. Acceleration is a vector. We use the symbol $\mathbf{a}$ to represent acceleration, and a to represent the magnitude.

You can accelerate an object by changing its speed, its direction of motion, or both its speed and direction. When you accelerate your car in a straight line from a stoplight, you are changing your speed. For example, suppose you are stopped, so that at time $t = 0$ seconds, your speed $v = 0$ km per hour; and in 6 seconds you increase your speed to 20 km per hour. These two data points are plotted on a (t, v) coordinate plane in Figures 3.5 and 3.6.

Table 3.2

Time, t	Speed, v
0 s	0 km/h
6 s	20 km/h

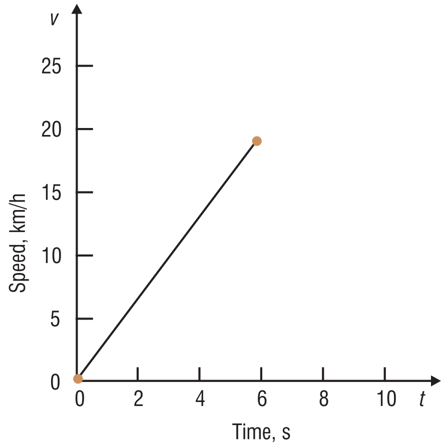


Figure 3.5
Uniform acceleration

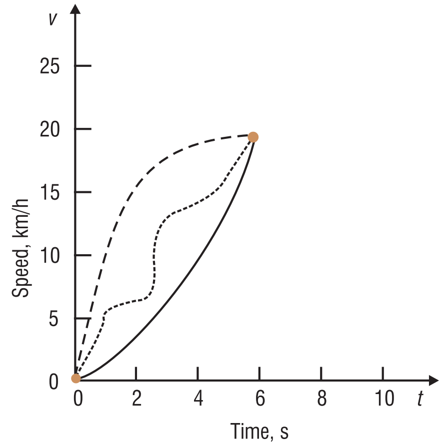


Figure 3.6
Nonuniform accelerations

In Figure 3.5, we have drawn the speed change as a constant. This is constant, or uniform, acceleration. Three other possible ways of changing speed are shown in Figure 3.6. These are nonuniform accelerations. Regardless of how the speed changes from 0 s to 6 s, the two values of speed (0 km/h and 20 km/h) can be used to calculate acceleration. But the ratio is the **average acceleration** between these times.

$$\text{Average acceleration} = \frac{\text{velocity change}}{\text{time interval}}$$

$$\mathbf{a}_{\text{ave}} = \frac{\Delta \mathbf{v}}{\Delta t}$$

For the data in Table 3.2, we can use magnitudes since the direction of the car's motion does not change. In this case, the acceleration is in the same direction as velocity.

$$\begin{aligned} a_{\text{ave}} &= \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} \\ &= \frac{20 \text{ km/h} - 0 \text{ km/h}}{6 \text{ s} - 0 \text{ s}} \\ &= \frac{20 \text{ km/h}}{6 \text{ s}} \\ &= 3.33 \text{ km/h per s} \end{aligned}$$

On average, during the six seconds of acceleration, the car's speed changes by 3.33 km/h each second.

Example 3.3 Acceleration of an Airliner

A pilot increases the takeoff speed of an airliner from 20 ft/s to 200 ft/s in 30 seconds. Find the magnitude of the average acceleration of the airliner.

Solution:

$$a_{\text{ave}} = \frac{\Delta v}{\Delta t} = \frac{200 \text{ ft/s} - 20 \text{ ft/s}}{30 \text{ s}}$$

$$= \frac{180 \text{ ft/s}}{30 \text{ s}}$$

$$= 6.0 \text{ ft/s /s} \quad \text{or} \quad 6.0 \text{ ft/s}^2$$

During the takeoff acceleration, the airliner's speed increases an average of 6.0 ft/s each second.

If the speed of an object is decreasing, Δv is negative and a is negative. In this case, negative acceleration means the object is slowing down. We call this *deceleration*.

Figure 3.7 shows the graph of speed versus time for a motorcycle as it moves from one stoplight to another. Section A of the graph shows the motorcycle's acceleration as it starts from rest and increases its speed in first gear. Section B shows deceleration as the gears are shifted from first to second. Sections C and D show acceleration and deceleration for second gear. For which gear is acceleration greater, first or second? How do you know?

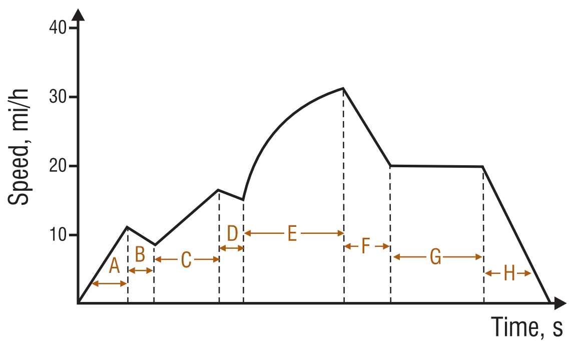


Figure 3.7
Speed versus time for a motorcycle

Section E shows acceleration in third gear. What is different about Section E compared to A and C? Section F shows deceleration as the motorcycle's brakes are applied.

What happens to the speed during Section G? What is the acceleration during this time?

Describe what is shown in Section H.

Angular Speed

Speed and velocity are rates of linear motion. *Angular speed* is a rate of rotational motion. An object such as a bicycle is in linear motion when it moves from one point to another along a straight line. The wheel of the bicycle also is in rotational motion. If you ride along on the bicycle, you will see the front wheel rotate. A point on the wheel's rim moves through an angle as the wheel rotates about its axis.

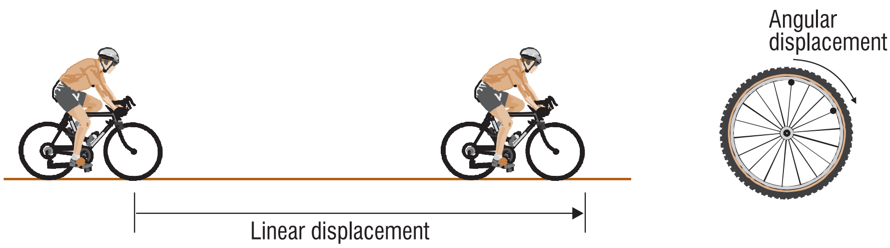


Figure 3.8
A moving bicycle is in linear motion.
To the rider, the front wheel is in rotational motion.

Linear displacement is the distance the bicycle moves to the right. Angular displacement is the angle through which the wheel rotates. In Figure 3.8, the wheel rotates clockwise. In this book, we will describe rotational motion as clockwise or counterclockwise. But we will not use vectors to describe rotational motion.

The bicycle's speed is the magnitude of linear displacement divided by the time interval. The wheel's **angular speed** ω is the angular displacement $\Delta\theta$ divided by the time interval Δt.

$$\text{Angular speed} = \frac{\text{angular displacement}}{\text{time interval}}$$

$$\omega = \frac{\Delta\theta}{\Delta t}$$

The symbol for angular speed ω is the Greek letter *omega*. The most common unit for angular speed is radians per second, where angular displacement Δθ is measured in radians. Recall from Section 2.1 that

$$1 \text{ revolution} = 360° = 2\pi \text{ radians}$$

Example 3.4 Angular Speed of a Second Hand

Calculate the angular speed of the second hand of a clock in radians per second.

Solution: The second hand makes one revolution in one minute. Therefore its angular speed is 1 revolution/min or 1 rpm.

$$\begin{aligned} \omega &= 1 \text{ rpm} \\ &= 1\frac{\text{rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{\text{rev}} \cdot \frac{1 \text{ min}}{60 \text{ s}} = 0.105 \text{ rad/s} \end{aligned}$$

The angular speed of the second hand of a clock is 0.105 radian per second.

Suppose the second hand of the clock in Example 3.4 is 10 cm long. What is the speed, in cm/s, of the tip of the second hand?

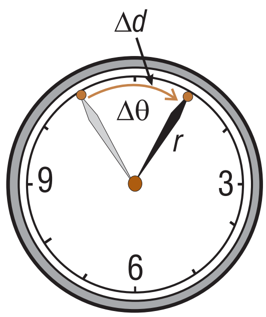


Figure 3.9

In an interval of time Δt, the second hand sweeps through an angular displacement Δθ. Let Δd represent the distance traveled by the tip of the second hand. Notice that Δd is a fraction of a circle's circumference. Remember that angular displacement measured in radians is

$$\Delta\theta = \frac{\Delta d}{r} \quad \text{and therefore} \quad \Delta d = r\Delta\theta$$

The speed is the ratio of distance traveled to the time interval.

$$v = \frac{\Delta d}{\Delta t} = \frac{r\Delta\theta}{\Delta t} = r\omega$$

The length of the second hand is the radius r.

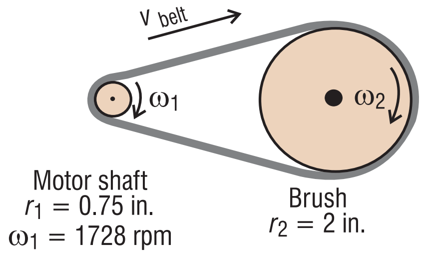
$$v = r\omega = (10 \text{ cm})\,(0.105 \text{ rad/s}) = 1.05 \text{ cm/s}$$

The speed of the second hand demonstrates a general relationship between speed v and angular speed ω:

> For any point on a rotating body, if the distance from the center of rotation to the point is r and the angular speed of rotation is ω, the speed v of the point is
>
> $$v = r\omega.$$

Example 3.5 Speed of a Vacuum Cleaner Belt

A vacuum cleaner motor shaft is 1.5 inches in diameter and turns at an angular speed of 1728 rpm. As the shaft turns, it moves a belt, which turns a brush. The brush is 4 inches in diameter.



A vacuum cleaner motor, belt, and brush assembly

What is the belt speed in inches per second? What is the angular speed of the brush, in revolutions per minute?

Solution: Let r_1 and ω_1 represent the radius and angular speed of the motor shaft. Convert units of ω_1 from rpm to radians per second:

$$\omega_1 = 1728 \frac{\text{rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{\text{rev}} \cdot \frac{1 \text{ min}}{60 \text{ s}} = 181 \text{ rad/s}$$

The belt moves with the same speed as a point on the outside circumference of the motor shaft. Let v_{belt} represent the speed of the belt.

$$v_{\text{belt}} = r_1\omega_1 = (0.75 \text{ in}) \, (181 \frac{\text{rad}}{\text{s}}) = 136 \text{ in/s}$$

The belt also moves with the same speed as a point on the outside circumference of the brush. Let r_2 and ω_2 represent the radius and angular speed of the brush.

$$v_{\text{belt}} = 136 \text{ in/s} = r_2\omega_2$$

$$= (2 \text{ in})\,\omega_2$$

$$\omega_2 = \frac{136\,\frac{\text{in}}{\text{s}}}{2 \text{ in}} = 68 \text{ rad/s}$$

Convert units to rpm:

$$\omega_2 = 68\,\frac{\text{rad}}{\text{s}} \cdot \frac{1 \text{ rev}}{2\pi \text{ rad}} \cdot \frac{60 \text{ s}}{1 \text{ min}} = 649 \text{ rev/min}$$

The speed of the belt is 136 in/s. The angular speed of the brush is 649 rpm.

Angular Acceleration

Have you ever seen an ice skater spin? He starts with his arms extended with an initial angular speed ω_1. As he brings his arms in closer to his chest, his angular speed increases, and reaches a final value ω_2. The change in angular speed is

$$\Delta\omega = \omega_2 - \omega_1.$$

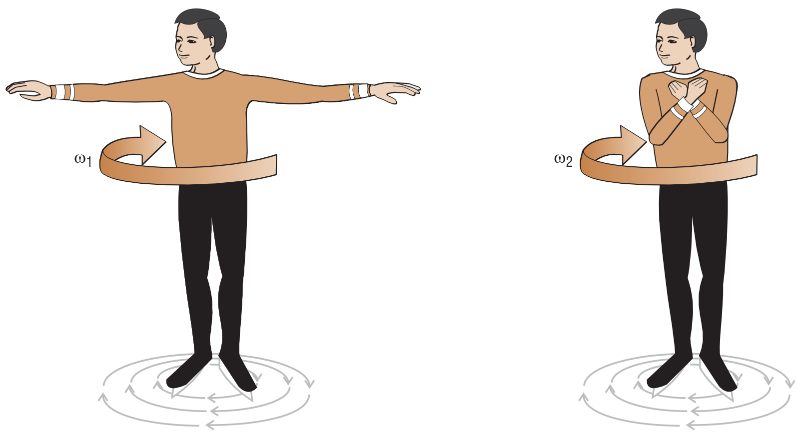


Figure 3.10
A skater increases his angular speed by bringing his arms in closer to his chest.

This change in angular speed occurs over a time interval Δt. The skater's **angular acceleration** α (the Greek letter *alpha*) is the ratio of the change in angular speed to the time interval. We will not use vectors to describe angular acceleration. In this book, angular acceleration will be treated as a scalar quantity.

$$\text{Angular acceleration} = \frac{\text{change in angular speed}}{\text{time interval}}$$

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

Example 3.6 Angular Acceleration of a Wheel

A car's brake is applied to a wheel for 5 seconds, reducing the wheel's angular speed from 220 rad/s to 180 rad/s. What is the angular acceleration?

Solution: The initial angular speed ω_1 is 220 rad/s, and the final angular speed ω_2 is 180 rad/s.

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_2 - \omega_1}{\Delta t} = \frac{180 \text{ rad/s} - 220 \text{ rad/s}}{5 \text{ s}}$$

$$\alpha = \frac{-40 \text{ rad/s}}{5 \text{ s}} = -8 \text{ rad/s}^2$$

The negative value of acceleration indicates that the wheel is slowing down—it is being decelerated. The wheel decelerates at a rate of 8 rad/s^2.

Summary

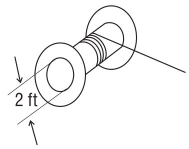
- Speed is a measure of the rate of motion of an object. It is the ratio of distance traveled to the time interval. Speed is a scalar quantity.
- Velocity is the ratio of displacement to the time interval. Velocity and displacement are vector quantities. Speed is the magnitude of velocity.
- Acceleration is a measure of the rate of change of an object's velocity. It is the ratio of change in velocity to the time interval.
- Angular speed is a measure of the rate of rotational motion of an object. It is the ratio of angular displacement to time interval.
- Angular acceleration is a measure of the rate of change of an object's angular speed. It is the ratio of change in angular speed to the time interval.

Exercises

1. What are the SI units for speed? Acceleration? Angular speed? Angular acceleration?
2. Explain the difference between speed and velocity.
3. Explain the difference between velocity and acceleration.
4. In qualifying for a race, a car completes one lap around a 1.5-mile track in 34.662 seconds.
   (a) What is the car's average speed in miles per hour?
   (b) What is the car's average velocity? Explain your answer.
5. An airplane's propeller completes one revolution in 65 ms. (ms = 10^{-3} s)
   (a) What is the angular speed in rpm? In rad/s?
   (b) The angular speed increases from the initial value (in part a) with an average acceleration of 7.3 rad/s^2 for 4.0 seconds. What is the propeller's final angular speed?
6. A printer head moves 15.6 cm from the left margin of the paper to the location where a line starts. The time required for the move is 0.030 second. What is the average speed of the printer head?
7. The Elco Voltmeter Company has a robot part-delivery system that moves at a speed of 3 ft/s. The path that the robot follows from the stock room through the assembly area is 570 feet in length. How much time is required for the robot to travel this path?
8. Speedlite Air Charter sends one of its airplanes from San Antonio to New Orleans to pick up Juanita and take her to Memphis, Tennessee, to a repair job. The plane flies 500 miles east to New Orleans in 1.81 hours. After picking up Juanita, the plane flies 370 miles north to Memphis in 1.41 hours.
   (a) What is the speed of the plane from San Antonio to New Orleans?
   (b) What is the speed of the plane from New Orleans to Memphis?
   (c) What is the speed of the plane for the combined flying time? Ignore any time on the ground in New Orleans.
   (d) What is the velocity of the plane for the complete trip? Ignore any time on the ground in New Orleans.
   (e) What is the magnitude of the velocity of the plane for the complete trip?

9. A motor turns 21.6 revolutions in 0.75 second. What is the angular speed of the motor in rpm? In radians per second?

10. A winch has a drum diameter of 2 feet. It turns at a rate of 3 rpm. How much cable will it pull in 5 minutes?



11. B.U.S. Chemical Company has a large compressor to compress hydrogen gas that is a by-product of its chlorine production. The compressor turns at 445 rpm.

    (a) What is the angular speed of the compressor in rev/s?

    (b) What is the angular speed of the compressor in rad/s?

12. The torque applied to the input shaft of the compressor in Exercise 11 is 2×10^4 N•m. How much work is done on the compressor in one second?

13. A Navy air-to-air missile is launched from an airplane with a speed of 1100 ft/s. Five seconds after launch the missile has a speed of 2460 ft/s. What is the average acceleration during the first five seconds of flight?

14. A semi-trailer truck has a speed of 60 mph and an engine speed of 1850 rpm. The driver accelerates to 65 mph in 26 seconds while the engine speed increases to 2005 rpm.

    (a) What is the linear acceleration of the truck in ft/s?

    (b) What is the angular acceleration of the engine in rpm/s?

    (c) What is the angular acceleration of the engine in rad/s^2?

15. The instantaneous linear speed of an object moving in a circle can be found from its angular speed and the radius of the circle, <u>if the angular speed is expressed in units of rad/s</u>.

    linear speed = (radius)(angular speed)

    A fan blade is turning at 345 rpm. The radius of the fan blade is 1.4 m. What is the linear speed of the tip of the fan blade?

16. A 150-ft-diameter Ferris wheel completes one revolution in 45 seconds.

    (a) How far does a chair on the Ferris wheel travel in one revolution?

    (b) What is the linear speed of the chair in ft/s?

    (c) What is the angular speed of the chair in rad/s?