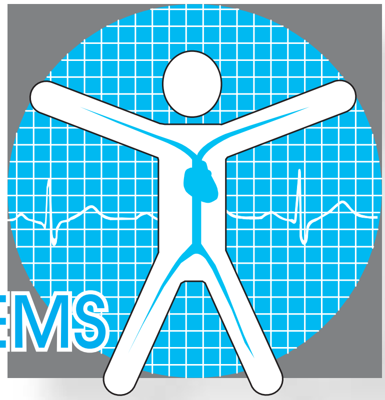
2.2 Work in Fluid Systems



Objectives

- Describe how open and closed fluid systems are different.
- Explain the relationship between work and pressure in a fluid system, as given in the equation,

$$W = P \times \Delta V$$

- Explain the relationship between work and pressure in a fluid system, as given in the equation,

$$W = -\Delta P \times V$$

- Explain what is meant by positive work and negative work.
- Solve work problems (given pressure and volume information) in English and SI units.

INTERNET *connection*

To find out more about work in fluid systems, follow the links at **www.learningincontext.com.**

In fluid systems, as in mechanical systems, work is done when a force moves an object through a distance. But it is easier to calculate work in a fluid system using the prime mover, pressure (or pressure difference).

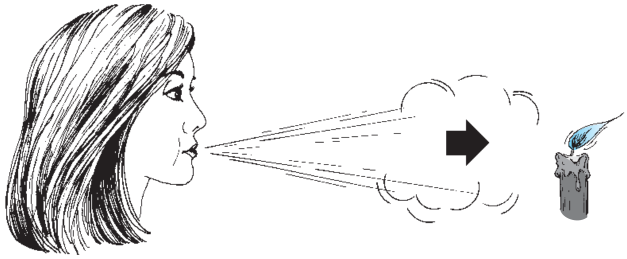
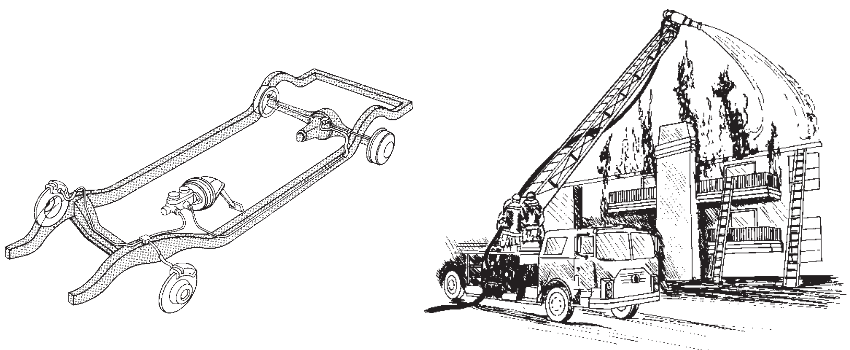


Figure 2.7
Pressure difference causes fluid motion.

A pressure difference can cause a fluid to move. For example, winds are caused by air moving from high-pressure regions to low-pressure regions. You create low- and high-pressure regions when you breathe, and these pressure differences cause air to move into and out of your lungs. When you inhale, you do work to expand your chest cavity and create a region of low pressure inside your lungs. A higher-pressure region outside then forces air into the lungs. When you exhale, you do work to shrink, or decrease, your lung volume. This increases the air pressure in your lungs and forces the air out.

Open and Closed Fluid Systems

We will analyze work in two types of fluid systems—open and closed. A **closed fluid system** retains and recirculates a working fluid. Examples of closed fluid systems are a hydraulic jack, a hydraulic brake system, and the body's circulatory (blood) system. Fluid flows through an **open fluid system** only one time—it is not retained and recirculated. Open fluid systems include city water systems, an irrigation system, and a fire truck water system.



(a) Brake system in car (closed)   (b) Fire truck (open)

Figure 2.8
Open and closed fluid systems

Hydraulic Actuator

Hydraulic actuators are used widely in industry and transportation. They control the movement of airplane landing gears, bulldozer shovels, and forklifts. A basic hydraulic actuator, shown schematically in Figures 2.9 and 2.10, is a closed system. It includes several subsystems, so it can be called a "system of systems." The actuator uses hydraulic fluid to exert pressure on a piston, and work is done when the piston moves a load.

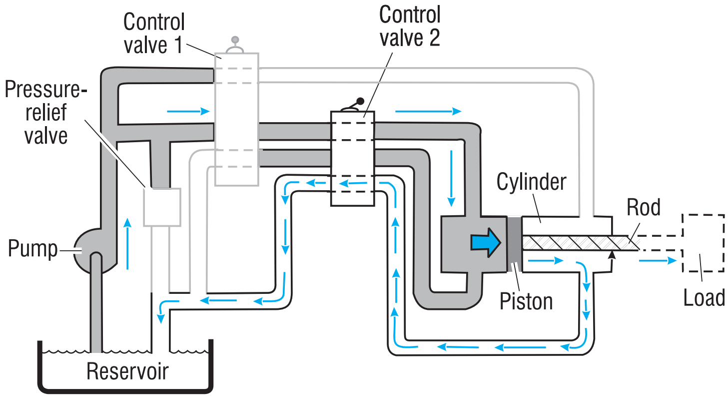


Figure 2.9
A basic hydraulic actuator. The pump moves fluid through control valve 2 into the left side of the cylinder. Control valve 1 is closed and does not allow fluid to flow through it. The piston moves to the right.

The pump draws hydraulic fluid from the reservoir, creates a pressure difference, and forces fluid through the pipes under pressure. The pump is driven by a rotating shaft that's connected to an electric motor. The control valves direct the fluid to the cylinder to either push or pull the load. When valve 1 is closed and valve 2 is open, high-pressure fluid flows to the left side of the cylinder. This creates a force on the piston that causes it to move, extending the rod and pushing the load. This is the arrangement shown in Figure 2.9.

When valve 1 is open and valve 2 is closed, high-pressure fluid flows to the right side of the piston. This causes the rod to retract, pulling the load. This arrangement is shown in Figure 2.10. With control valves in either setting, as the piston in the cylinder moves, fluid is forced out of the cylinder and directed back to the reservoir.

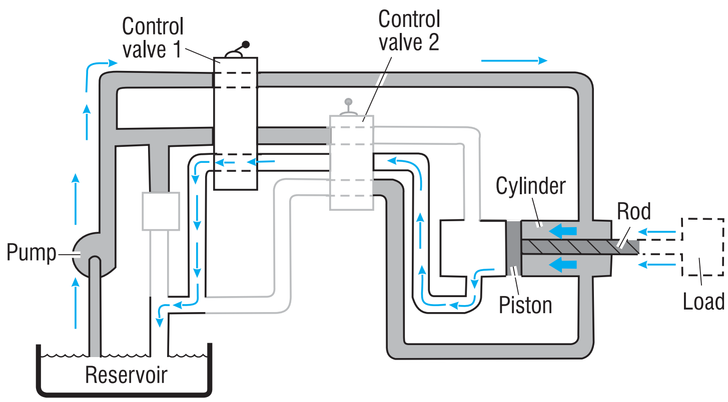


Figure 2.10
A basic hydraulic actuator. Fluid flows through control valve 1 into the right side of the cylinder. Control valve 2 is closed and does not allow fluid to flow through it. The piston moves to the left.

Refer to Appendix F for a career link to this concept.

Work Done in Fluid Systems

Figure 2.11 shows a simple closed fluid system—a cylinder filled with a gas and enclosed by a movable piston. The piston has a fixed weight on top to maintain a constant pressure inside the cylinder.

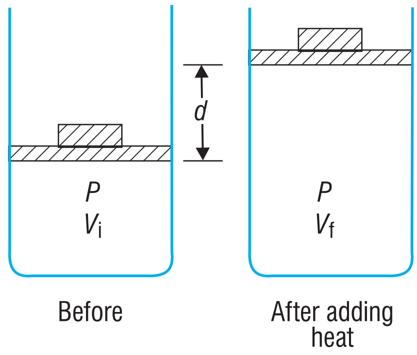


Figure 2.11
A cylinder and piston make a simple closed fluid system. The piston is one boundary of the system, and this boundary can move.

The volume of the gas in the cylinder is originally V_i. When heat is added to the gas in the cylinder, the gas expands to a volume V_f. The change in volume is $\Delta V = V_f - V_i$. The piston rises a distance d from its original position. The cross-sectional area A of the cylinder does not change, so the volume change ΔV is

$$\Delta V = A \times d$$

The work W done by the fluid in lifting the piston is the force times the distance moved in the direction of the force. The force F is the pressure P times the area A. The pressure does not change because the weight supported by the piston does not change.

$$\begin{aligned} F &= P \times A \\ W &= F \times d \\ &= (P \times A) \times d \\ &= P \times (A \times d) \\ &= P \times \Delta V \end{aligned}$$

This is the equation for calculating work done by a fluid on the moving boundary of a closed system when the pressure is constant and the volume changes.

$$\text{Work} = \begin{matrix}\text{constant}\\ \text{pressure}\end{matrix} \times \begin{matrix}\text{fluid volume}\\ \text{change}\end{matrix}$$

$$W = P \times \Delta V$$

Notice that ΔV can be positive or negative. (P is always positive.)

- If the fluid volume increases, ΔV is positive and W is positive. Positive work means *the fluid does work*, as when a gas expands in a cylinder, lifting a load.
- If the fluid volume decreases, ΔV is negative and W is negative. A negative value for work means that *work is done on the fluid*. For example, a weight or force applied to the piston compresses the gas in a cylinder. This applied force does work on the gas.

Figure 2.12 shows a simple open fluid system—a water pump. Water enters the pump at pressure P_i. The pump does work on the water and increases its pressure to P_f. In a given time interval, the same amount of water must enter and leave the pump. (There is no source of extra water, and there is no place to store water inside the pump.) This is called a *steady-flow* process. If 1 kg of water per second enters the pump, 1 kg per second must leave the pump.

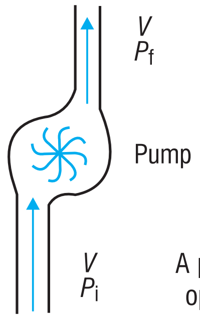


Figure 2.12
A pump is a simple open fluid system.

In one second, the pump will do work on 1 kg of water. If the density is constant, the volume moved is $V = m/\rho = \dfrac{1 \text{ kg}}{10^3 \text{ kg/m}^3} = 10^{-3} \text{ m}^3$.

The following equation is used for calculating work done by a fluid in a steady-flow process when the density is constant and the pressure changes.

$$\text{Work} = \text{pressure difference} \times \text{volume moved}$$

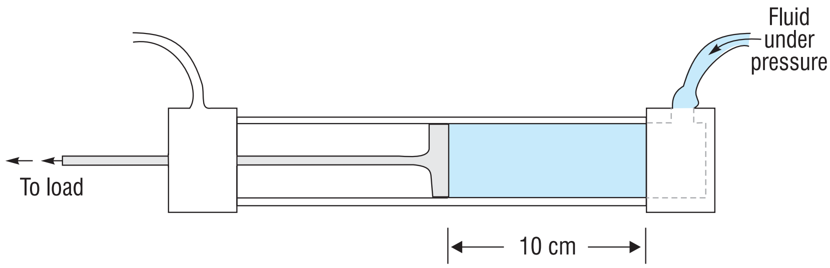
$$W = -\Delta P \times V$$

The negative sign in the equation is important. The pressure difference ΔP can be positive or negative. (V is always positive.)

- If the pressure increases, ΔP is positive and W is negative. As before, a negative value for work means that *work is done on the fluid*. A pump is a typical source of work done on a fluid.
- If the pressure decreases, ΔP is negative and W is positive. Positive work means *the fluid does work*. In a hydroelectric dam, water flows from a high-pressure region behind the dam to a low-pressure region, turns a turbine, and does work.

Example 2.6 Work Done by a Hydraulic Cylinder

An industrial robot moves objects with hydraulic cylinders. Each cylinder contains a piston and a working fluid (oil). The oil is forced into the cylinder under high pressure on one side of the piston. The oil pushes against the piston. This causes the piston to move. The other end of the piston is connected to a load. As the piston moves, it moves the load.



The fluid is injected into the cylinder at a constant pressure of 80 kPa. The cross-sectional area of the cylinder is 12 cm^2. Find the work done by the hydraulic fluid while moving the piston and load 10 cm.

Solution: The pressure on the piston is constant, and the volume of oil in the cylinder changes.

$$\text{Work} = \begin{matrix}\text{constant}\\\text{pressure}\end{matrix} \times \begin{matrix}\text{fluid volume}\\\text{change}\end{matrix}$$

$$W = P \times \Delta V$$

No oil is in the cylinder initially. Therefore, $V_i = 0\ cm^3$. The volume of oil that enters the cylinder and moves the piston is the same as the volume of a cylinder of cross-sectional area 12 cm^2 and length 10 cm.

$$V_f = 12\ cm^2 \times 10\ cm = 120\ cm^3$$

The change in volume is $\Delta V = V_f - V_i = 120\ cm^3$.

Convert to m^3:

$$\Delta V = 120\ cm^3 \times \frac{1\ m^3}{10^6\ cm^3} = 1.2 \times 10^{-4}\ m^3$$

$$W = P \times \Delta V$$

$$= 80\ kPa \times (1.2 \times 10^{-4}\ m^3)$$

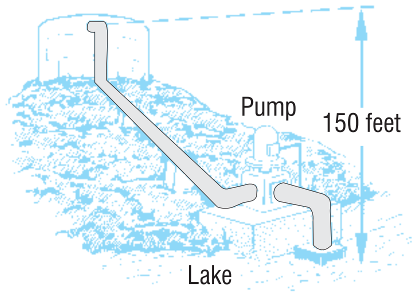
$$= (8.0 \times 10^4\ N/m^2) \times (1.2 \times 10^{-4}\ m^3) \qquad [Pa = N/m^2]$$

$$= 9.6\ N{\cdot}m \quad \text{or} \quad 9.6\ J$$

The hydraulic fluid does 9.6 J of work while moving the load attached to the piston a distance of 10 cm.

Example 2.7 Work Done by a Pump to Fill a Tank

A community's water supply is stored in an elevated tank to maintain water pressure. The tank is filled by water pumped from a reservoir 150 feet below the inlet to the tank. The tank has a volume of 5000 ft^3. If the tank is empty, what minimum work must be done by the pump to fill it?



Solution: The volume of water moved by the pump is 5000 ft^3. The pump increases the water pressure. The minimum work done by the pump is that required to push the water up the 150-ft height.

$$\text{Work} = \begin{matrix}\text{pressure} \\ \text{difference}\end{matrix} \times \begin{matrix}\text{volume} \\ \text{moved}\end{matrix}$$

$$W = -\Delta P \times V$$

Find the pressure difference for this situation. On the low-pressure side of the pump (the lake side), the water pressure P_i equals atmospheric pressure P_{atm}. On the high-pressure side of the pump (the tank side), the water pressure P_f equals P_{atm} plus the pressure of a column of water 150 feet high. The weight density ρ_w of water is 62.4 lb/ft^3.

$$P_i = P_{atm}$$

$$\begin{aligned} P_f &= P_{atm} + \rho_w h \\ &= P_{atm} + 62.4\frac{\text{lb}}{\text{ft}^3} \times 150 \text{ ft} \\ &= P_{atm} + 9360 \text{ lb/ft}^2 \end{aligned}$$

$$\begin{aligned} \Delta P &= P_f - P_i \\ &= (P_{atm} + 9360 \text{ lb/ft}^2) - P_{atm} \\ &= 9360 \text{ lb/ft}^2 \\ W &= -\Delta P \times V \\ &= -9360 \text{ lb/ft}^2 \times 5000 \text{ ft}^3 \\ &= -4.68 \times 10^7 \text{ ft}\cdot\text{lb} \end{aligned}$$

The negative sign means that work is done on the fluid. The pump must do at least 4.68×10^7 ft·lb of work to fill the tank.

Other Examples of Work in Fluid Systems

You've studied work done in a closed fluid system (hydraulic cylinder) and an open fluid system (pumping water to fill a tank). You found fluid work to be positive when the fluid does work and negative when work is done on the fluid.

Other fluid systems that do work include the following:

- Burning fuel-air mixture in a cylinder of an internal-combustion engine (*Positive work* is done by the expanding gas.)
- A heart pumping blood (The blood does *negative work*—the heart does work on the blood.)
- A refrigerator compressor using a piston and cylinder to increase the temperature and pressure of the working fluid (The fluid does *negative work*—the piston does work on the fluid.)
- Burning fuel expanding through a nozzle to lift a rocket (*Positive work* is done by the fuel.)
- Pushing down on the input cylinder of a hydraulic jack (The hydraulic fluid does *negative work*—work is done on the fluid.) and the output cylinder lifting a heavy load (*Positive work* is done by the fluid.)

If you look back at the basic hydraulic actuator shown in Figures 2.9 and 2.10, you can see that three different forms of work are involved in the operation of the "system of systems." Electrical work is done on a motor (not shown) operating the pump. The pump does mechanical (rotational) work to create a pressure difference for the fluid. The fluid, under pressure, does work on the piston and load. It's often true, as shown in this system, that work is being done at several places at the same time. That's why understanding work in mechanical, fluid, and electrical systems is important.

You can now calculate work in mechanical and fluid systems. Notice the similarity in the equations. Work equals the prime mover times another quantity:

In mechanical systems:	Work $= F \times d$	or	Work $= \tau \times \theta$
In fluid systems:	Work $= P \times \Delta V$	or	Work $= -\Delta P \times V$

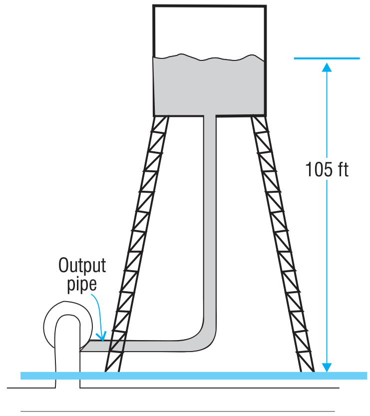
Summary

- Fluids flow from regions of high pressure to regions of low pressure.
- Closed fluid systems retain and recirculate fluids.
- Open fluid systems move fluids into and out of the systems. They don't retain or recirculate fluids.
- A fluid does work in a closed system when a boundary moves. The pressure is constant and the volume changes, $W = P \times \Delta V$.
- A fluid does work in a steady-flow, constant-density process when a fixed volume moves through a pressure difference. $W = -\Delta P \times V$.
- Fluid work can be positive or negative.

Exercises

1. When a fluid does positive work in a closed system and the pressure is constant, does the volume of the fluid increase or decrease?
2. When a fluid does positive work in a steady-flow process and the volume is constant, does the pressure of the fluid increase or decrease?
3. Why is a pump an *open* fluid system?
4. Why do a cylinder and piston form a *closed* fluid system?
5. For each of the following situations, tell if the air does positive or negative work.
   (a) A sailboat moves forward in the wind.
   (b) You move air out of your lungs when you exhale.
   (c) A fan draws air through a car's radiator.
   (d) A balloon flies around the room as it deflates.
6. Tell whether each of the following is true or false. If a statement is false, explain why.
   (a) In a closed fluid system, work is always positive.
   (b) If the pressure and volume of a fluid are both constant, the fluid does no work.
   (c) If fluid work is negative, work is done on the fluid.
   (d) In a steady-flow process, the final pressure is always greater than the initial pressure.

A pump is used to maintain the water level in a tank at 105 feet above the pump, as shown at the right. Use this information to answer Exercises 7–9.

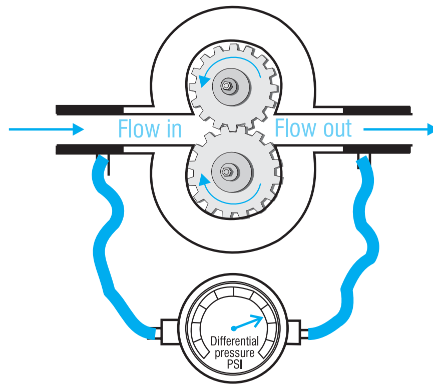


7. When the pump is not running, what is the water pressure at the output pipe of the pump?

8. To move water, the pump must overcome friction between the water and the pipe. If the pressure required to overcome friction is 5 psi, how much pressure, in psi, must the pump supply to pump water into the tank?

9. How much work must the pump do to put 120 ft^3 of water in the tank?

10. A hydraulic motor has a *displacement* of 2.5 in^3. This means that, each time the shaft makes one revolution, 2.5 in^3 of hydraulic fluid go through the motor. Louis measures a pressure drop of 75 psi across the motor while it is in use. He measures the motor speed to be 120 rpm. How much work, in ft·lb, does the fluid do in one minute? How much work, in ft·lb, does the fluid do in one second?



11. The amount of work done by a motor in one second is called the **power** of the motor. In English units, power can be expressed in foot-pounds/second or in **horsepower**.

$$1 \text{ horsepower} = 550 \text{ ft·lb/s}$$

What is the power produced by the hydraulic motor in Exercise 10, expressed in horsepower?

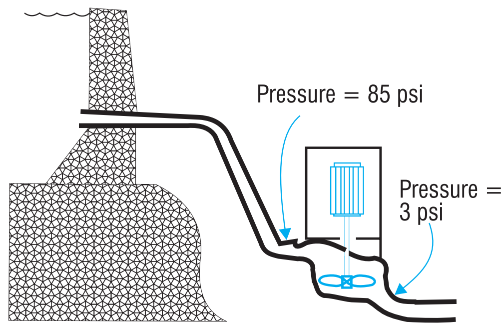
12. A robot arm is actuated by hydraulic cylinders. The arm is used to lift a 53.8-pound truck door 4.2 feet and hold it in place while the hinge bolts are installed. A volume of 28 in^3 of hydraulic fluid is pumped into a hydraulic cylinder while it lifts the door.

(a) What work must be done to lift the door?

(b) What is the pressure of the fluid pumped into the cylinder?

13. The diameter of an uninflated, spherical balloon is 3.0 cm. To inflate the balloon, your lungs provide a constant pressure with each breath, equal to 1.2 kPa over atmospheric pressure. The diameter of the inflated balloon is 30 cm. Calculate the work done by your lungs to inflate the balloon. (The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.)

14. A small hydroelectric plant has an input water pressure of 85 psi and an output pressure of 3 psi. Note that the direction of the water flow is against the pressure drop when the water is causing the turbine to turn. How much work is done by the water when 8500 gallons flow through the turbine? (1 gal = 231 in^3)



15. The air-conditioning unit in the Lake Air High School auditorium is being replaced. The building code requires that the system be capable of moving enough air to change the air in the auditorium 6 times per hour. The dimensions of the auditorium are 75 ft by 110 ft by 16 ft. A fan produces the pressure difference required to move the air through the filters and ductwork. The required pressure difference is 0.1 psi.

(a) Convert the pressure difference to lb/ft^2.

(b) How much work, in ft·lb, is done by the fan on the air in one hour?

(c) How much work is done by the fan in one second?

(d) What is the power absorbed by the air-circulation system expressed in horsepower? (See Exercise 11.)