2.1 WORK IN MECHANICAL SYSTEMS

Objectives

- Define work done by a force or torque in a mechanical system.

- Explain the relationship between work, force applied, and the distance an object moves.

- Solve work problems, given force and distance information in English and SI units.

- Explain how efficiency relates to input work and output work for a mechanical system.

- Define radian measure of angles.

- Explain the relationship between work, torque applied, and the angle (in radians) through which an object moves.

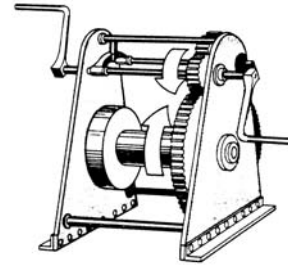- Solve work problems, given torque and angle information in English and SI units.

In mechanical systems, work is done when a force moves an object through a distance. An electric chain hoist does work as it raises a beam. The hoist exerts a force to overcome the weight of the beam, and this force acts over the distance through which the beam moves. In meshing gears, one gear drives another—the driving gear does work on the driven gear. You will see in this section that it is easier to use torques than forces when calculating work in rotating systems.
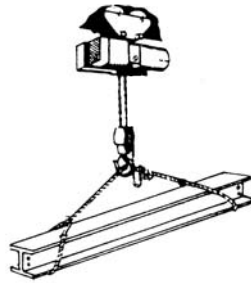
(a) Chain hoist raises a load.   (b) Gear turns a shaft.

Mechanical systems do work when forces act through distances.

Work Done by a Force

The **work** W done on an object is defined as the product of the applied force F and the distance d through which the object moves.

$$\text{Work} \ = \ \text{force} \times \ \begin{array}{l} \text{distance moved in the} \\ \text{direction of the force} \end{array}$$

$$W \ = \ F \times \boldsymbol{d}$$

The units for work come from multiplying a force (in newtons or pounds) by a distance (in meters or feet). Thus, in SI, work is measured in **newton-meters** (N·m) or **joules** (J). 1 N·m = 1 J. In the English system, work is measured in **foot-pounds**. The distance and direction through which an object moves are also called its *displacement*.

The definition of work uses the magnitudes of force and displacement. These quantities are actually vectors, since they also have direction. But we will consider only situations in which force and displacement are in the *same* direction or in *opposite* directions. Thus, work can be positive or negative. If the force on an object and the displacement are in the same direction, the work on the object is *positive*. If the force and the displacement are in opposite directions, the work is *negative*.
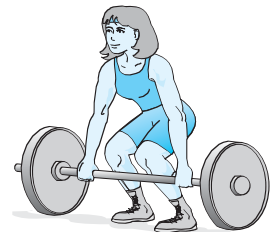
Example 2.1 Work Done by a Weight Lifter

Find the amount of work done by a weight lifter in lifting a 200-lb barbell a distance of 5 ft.

Solution: $W = F \times d$

$= (200 \text{ lb}) (5\text{ft})$

$= 1000 \text{ ft·lb}$

The weight lifter does 1000 ft·lb of work.

Now let's take a closer look at the process of lifting a barbell to see exactly what happens (see Figure 2.2).

Suppose you're going to lift the 200-pound barbell. At first, when the barbell is at rest on the floor, the forces on the barbell are balanced. Gravity pulls the barbell down with a force of 200 pounds (its weight) while the floor pushes up with an equal and opposite force of 200 pounds. No motion occurs; therefore, no work is done.

As you begin to lift the barbell, your upward pull is a little stronger than the downward pull of gravity. The barbell begins to move upward. You're now doing work on the barbell. When the barbell is moving, a force of 200 pounds will keep it moving upward *at a constant speed*. This 200-pound upward force does work on the barbell as it moves to its highest position, 5 feet from the floor.

| Barbell is not moving. No work is done. | Work is done while barbell is moving. | Barbell is not moving. No work is done. |
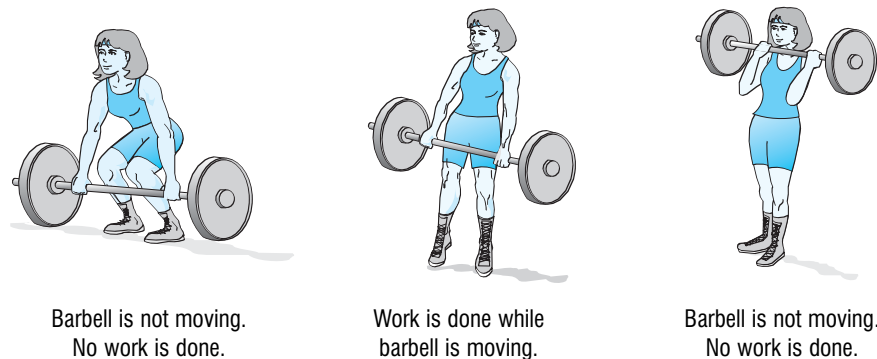
Figure 2.2
A closer look at lifting a barbell

When you've lifted the barbell to five feet, the barbell stops moving. You must still exert a force of 200 pounds to hold it in this position. But this force does no work on the barbell, since the barbell does not move. You will become tired because work is still done by your muscles as they stretch and contract small amounts to keep the barbell in place. These small forces cause small muscular movements, and do work, but this "biological work" is not being done on the barbell.

Example 2.2 Work Done to Stop a Cart

An electric cart is moving to the right. The driver applies the brakes, and stops the cart in 5 meters. A constant braking force of 900 newtons is applied over this distance. What is the direction of the braking force? How much work is done on the cart by this force?

Solution: To slow the cart, the braking force must act in a direction opposing the cart's motion. Therefore, the braking force acts to the left.
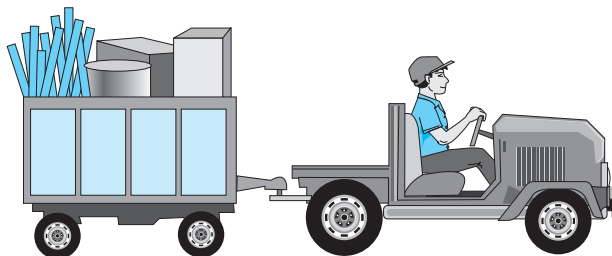
While the braking force is applied, the cart moves to the right. Thus, the force and distance moved are in opposite directions and the work is negative:

$$W = F \times d$$

$$= -(900 \text{ N}) \, (5 \text{ m})$$

$$= -4500 \text{ N} \cdot \text{m} \quad \text{or} \quad -4500 \text{ J}$$

The brakes do –4500 J of work on the cart.

In many situations, a force can start an object moving and the object's motion continues after the force is removed. For example, when you throw a ball, your arm applies a force over a small distance. But the ball's motion continues over a much longer distance. In cases like this, it's important to know that work is done only while the force is applied. This means that the distance used to calculate work is the distance the object moves while the force is applied. When you throw a ball, your arm does not apply a force on the ball after it leaves your hand. So, even if the ball keeps moving, you do no work after the ball leaves your hand.

Work Can Change an Object's Potential Energy or Kinetic Energy

Example 2.1 shows that doing work on an object can change its position in the Earth's gravitational field. You learned in Section 1.4 that **potential energy** is the form of energy stored by an object. Potential energy may be due to an elevated position of a body. Thus, when the weight lifter does work on the barbell, she increases its gravitational potential energy. The amount of gravitational potential energy gained by the barbell equals the work done in lifting it. In other words, work equals change in energy.

Example 2.2 shows that doing work on an object can change its speed. You learned in Section 1.4 that **kinetic energy** is energy of motion. When the electric cart's brakes do work on the cart, the speed of the cart and its kinetic energy decrease. The amount of kinetic energy lost by the cart equals the work done on it by the braking force. Once again, work equals change in energy.

Work and Efficiency

Machines are designed to convert energy or work *input* to useful forms of energy or work *output*. In real (nonideal) machines, output work is always less than input work because some input work must be done to overcome undesired but unavoidable losses. For example, whenever two moving parts of a machine are in contact, a force of friction opposes the motion. This force does negative work, and therefore represents work lost by the machine.

When machines do work, they're often rated by efficiency. Efficiency is a ratio of output work to input work. Output is the work done by the machine. Input is work done on the machine to make it operate. In real machines, output work is always less than input work, so efficiency is always less than 1, or 100%.

$$\text{Efficiency} = \frac{\text{output work}}{\text{input work}}$$

$$\text{Percent efficiency} = \frac{\text{output work}}{\text{input work}} \times 100\%$$

A block and tackle is a simple machine consisting of a cord and a system of pulleys used to lift a heavy load. The upper set of pulleys is securely attached at one point. The lower set moves with the load.
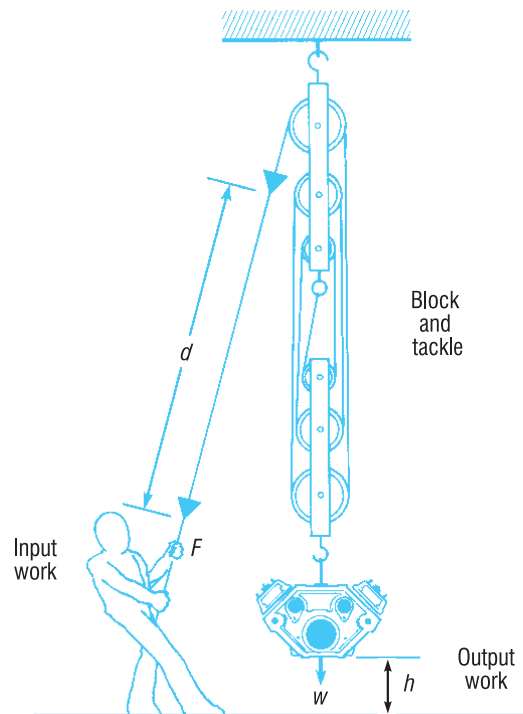
Figure 2.3
With a block and tackle, a force *F* can lift a larger weight *w*.

A worker pulls the free end of the cord with force F. This force moves the cord through a distance d. At the same time, a load of weight w is raised through distance h by the action of the block and tackle.

The output work is the work done by the block and tackle to raise the load. The input work is the work done by the force F that causes the block and tackle to operate. Since work is equal to force times distance, the input work is equal to $F \times d$ and the output work is equal to $w \times h$. Friction in the bearings of the pulleys will cause output work to be less than input work.

Example 2.3 Work and Efficiency for a Block and Tackle

The block and tackle shown in Figure 2.3 is used to lift an automobile engine. The engine weighs 600 lb and is raised 0.9 ft. The operator pulls with a force of 100 lb over a distance of 6 ft. Calculate the efficiency of the block and tackle system.

Solution:

$$\text{Input work} = F \times d \qquad\qquad \text{Output work} = w \times h$$

$$= (100\,\text{lb})\,(6\,\text{ft}) \qquad\qquad\qquad = (600\,\text{lb})(0.9\,\text{ft})$$

$$= 600\,\text{ft}\cdot\text{lb} \qquad\qquad\qquad\qquad = 540\,\text{ft}\cdot\text{lb}$$

$$\text{Efficiency} = \frac{\text{output work}}{\text{input work}}$$

$$= \frac{540\,\text{ft}\cdot\text{lb}}{600\,\text{ft}\cdot\text{lb}}$$

$$= 0.9 \text{ or } 90\%$$

The block-and-tackle system is 90% efficient.

How much of the 600 ft·lb of input work is lost to friction in the block and tackle?

Measuring Angles in Radians

In Section 1.1, you learned that, when a force is applied to an object at some distance away from an axis of rotation, the resulting torque can cause the object to rotate. If the object rotates, the applied force acts through a distance (in the direction of movement) and does work on the rotating object. Instead of using force and distance, it is easier to use torque and the angle of rotation to calculate work for rotating objects.

First, you need to know how to measure angles in units called **radians**. A radian is defined by placing the vertex of the angle to be measured at the center of a circle. The sides of the angle intersect the circle at two points. Measure the length of the arc of the circle between these points. Arc length is a fraction of the circumference of the circle. The measure of the angle in radians is the arc length divided by the radius of the circle.
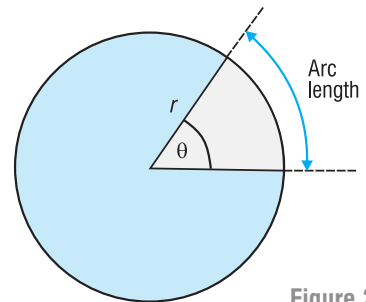
Figure 2.4
Angle measure in radians
$$\theta = \frac{\text{Arc length}}{r}.$$

$$\text{Angle measure in radians} = \frac{\text{arc length}}{\text{radius}}$$

$$\theta = \frac{\text{arc length}}{r}$$

Remember, the circumference of a circle of radius r equals $2\pi r$. When an object completes one revolution (1 rev), it rotates through 360°. This is a whole circle, so the arc length is the whole circumference:
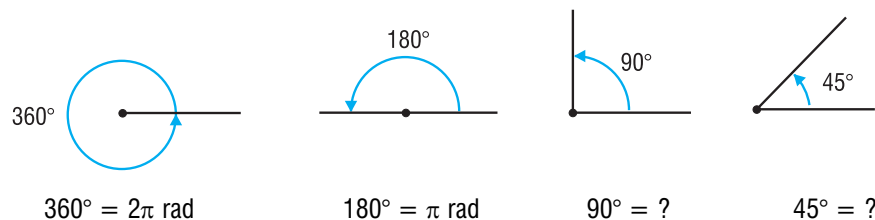
$$1 \text{ rev} = 360° = \frac{\text{arc length}}{r} = \frac{2\pi r}{r} = 2\pi \text{ rad}$$

The abbreviation for radian is *rad*. Since $\pi \approx 3.14$, $360° \approx 6.28$ rad.

A 180° angle is a half circle, so the arc length is half the circumference:

$$180° = \frac{\frac{1}{2}(2\pi r)}{r} = \pi \text{ rad}$$

Can you show how to convert 90° and 45° to radians?

360° = 2π rad   180° = π rad   90° = ?   45° = ?

Figure 2.5
Angles in degrees and radians

The radian is an odd kind of unit. The measure of an angle in radians is a ratio of lengths, and is therefore a pure number with no dimensions. Don't forget, in solving problems mathematically, you must be careful to solve for the correct units—as well as the correct number. We drop the unit *rad* in a calculation unless the result is an angular quantity.

Work Done by a Torque

Figure 2.6 shows a wrench turning a nut. A force F acts on the wrench at a distance r from the axis of rotation. The force produces a torque τ. These quantities are related by

$$\tau = F \times r \quad \text{or} \quad F = \frac{\tau}{r}$$

If the wrench rotates, the force does work. The work equals the force times the distance through which the force moves. Since the movement of the wrench is circular, this distance is the arc length.
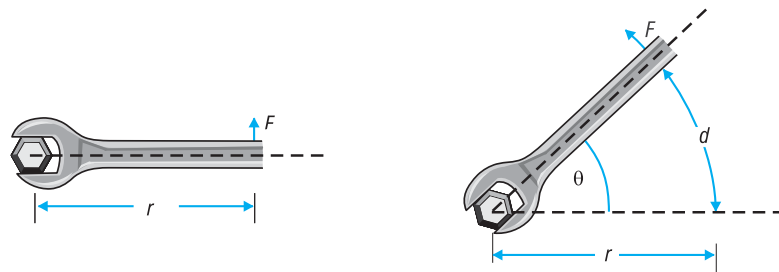

Figure 2.6
A force F produces torque. If the wrench rotates, F does work.

Let θ (the Greek letter *theta*) represent the measure of the angle of rotation.

If θ is measured in radians, $\theta = \frac{d}{r}$. What is the relationship between work W and torque τ?

$$W = F \times d$$

$$= \frac{\tau}{r} \times d$$

$$= \tau \times \frac{d}{r}$$

$$= \tau \times \theta$$

The work done when a torque causes a rotation is given by the following form of the work equation:

$$\text{Work} = \text{torque} \times \begin{array}{l} \text{angle (in radians)} \\ \text{moved through} \end{array}$$

$$W = \tau \times \theta$$

A *winch* is a reel, a type of axle, with a crank to turn it. A winch is used to wind or unwind rope, wire, or cable. A boat trailer may have a winch. The winch is used to pull the boat out of the water and onto the trailer.

Example 2.4 Work Done to Turn a Crank

A simple mechanical winch has a crank handle of length 1.5 ft. A force of 20 lb is required to turn the crank. How much work is required to turn the crank through five revolutions?

$\tau = F \times r$
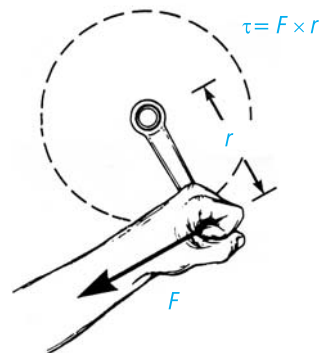
Solution: Find the torque:

$$\tau = F \times r$$
$$= (20 \text{ lb})(1.5 \text{ ft})$$
$$= 30 \text{ lb} \cdot \text{ft}$$

Convert the total angle of rotation, 5 revolutions, to radians.

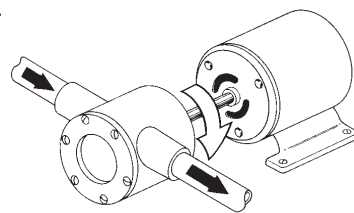$$\theta = (5 \text{ rev})\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)$$
$$= 10\pi \text{ rad}$$

Find the work done by the torque:

$$W = \tau \times \theta$$
$$= (30 \text{ lb} \cdot \text{ft})(10\pi)$$
$$= 944 \text{ lb} \cdot \text{ft} \quad \text{or} \quad 944 \text{ ft} \cdot \text{lb}$$

A work input of 944 ft·lb is required to turn the crank. Notice that units for work are written as ft·lb. Units for torque are written as lb·ft.

Example 2.5 Work Done to Operate a Pump

An electric motor is used to drive a water pump. The motor provides a torque of 150 N·m. How much work is done when the motor turns through 40 revolutions?

$$\theta = (40 \text{ rev})\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)$$
$$= 80\pi \text{ rad}$$
$$W = \tau \times \theta$$
$$= (150 \text{ N} \cdot \text{m})(80\pi)$$
$$= (150 \text{ N})(80\pi)$$
$$= 37,680 \text{ N} \cdot \text{m} \quad \text{or} \quad 37,680 \text{ J}$$

Therefore, 37,680 joules of work are done by the applied torque.

Summary

- Mechanical systems use force and torque to cause desired movement—and do useful work.

- Work is done when a force or torque moves an object. Work is done only while the force or torque is applied in the direction of movement.

- Work equals force times displacement or torque times angle. Work is measured in ft·lb or N·m (J = N·m). ($W = Fd$; $W = \tau\theta$)

- The displacement used to calculate work is the distance the object moves while the force is applied.

- Efficiency describes how well a machine performs work. Efficiency is the ratio of output work to input work. ($\text{Eff} = W_{out}/W_{in}$)

- Angles can be measured in either radians or degrees. The radian is a dimensionless unit and is used in most calculations involving angles.

Exercises

1. Positive mechanical work is done when a force acts on an object and the object _____ in the same direction as the force vector.

2. Positive mechanical work is done when a _____ acts on an object and it rotates in the direction of the _____ .

3. Negative mechanical work is done by a force on an object if the object moves in the direction *opposite* the force vector. For example, Eddy applies an upward force and lifts a 20-lb box to a height of 3 feet. Eddy does positive work of 60 ft·lb on the box. Gravity exerts a force down while the box is moving up. The gravitational force does work, but it is negative work. The gravitational force does –60 ft·lb of work.

   Find the work done in each of the following cases and determine whether it is positive or negative.

   (a) A parachute exerts a force of 5000 newtons on an object that falls 500 meters.

   (b) Larry pushes a cart up a ramp that is 12 meters long. He applies a force of 200 newtons parallel to the ramp.

   (c) A freight elevator carries a 1500-lb box down 300 feet. Find the work done by the elevator on the box.

   (d) The braking system on a car applies a force of 7200 lb and brings the car to a stop in 800 feet.

4. Why do frictional forces reduce the efficiency of machines?

5. When you use an angle measurement in a calculation and the units of the angle are radians, you can drop the rad. Explain why this does not affect the calculation.

6. The work unit of one joule is also equal to one _____.

7. In one or two sentences, explain what mechanical work does to an object.

8. An angle measuring one radian measures _____ degrees.

9. A man who weighs 180 pounds is lifted 200 feet by an elevator. How much work does the elevator do on the man? How much work does the gravitational force do on the man?

10. An automotive technician applies a torque of 120 lb·ft to tighten a cylinder head bolt. The bolt turns $1/8$ of a revolution. How much work is done on the bolt?

11. A gear motor applies a torque of 15 N·m to raise the landing gear of an airplane. The gear motor makes 27.3 revolutions. How much work is done by the gear motor?

12. An electric motor can produce a torque of 51.8 N·m and rotates at a speed of 1728 revolutions per minute. How much work can the motor do in one second?

13. A hydraulic motor rotates 28,000 revolutions to do 9×10^6 joules of work. What is the average torque produced by the motor?

14. A robotic delivery cart carries a part with a mass of 5.4 kg a horizontal distance of 12 meters on a flat floor. The wheels have a diameter of 20 cm and the drive motor applies a torque to the wheels of 5 N·m while the cart is moving.

    (a) How much work does the drive motor do?

    (b) How much work does the gravitational force do?

15. An electric motor on a forklift truck turns at a speed of 600 revolutions per minute (rpm) for 4 seconds. It produces a torque of 20 N·m to turn a hydraulic pump. The hydraulic system lifts a 154-kg box a vertical distance of 2.5 m. What is the efficiency of the system?
    (Hint: θ = angular rate × time)

16. Ricardo uses a chain hoist to lift a 645-pound air compressor 4.5 feet. Ricardo has to pull 54 feet of chain through the chain hoist. If the hoist is 95% efficient, how much force does Ricardo have to apply to the chain?